# Comparison of Efficiency of Severity Estimators Based on Different Data Aggregation Levels


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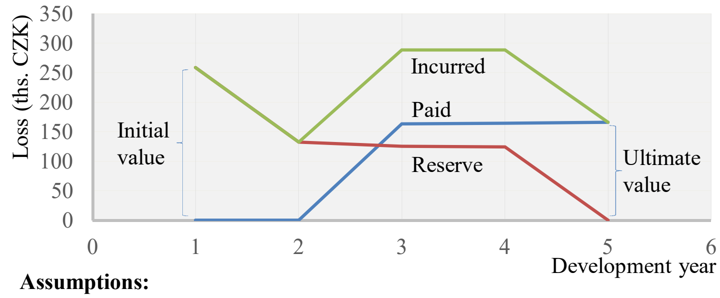
**Main tasks:** ■ Definition of a 'micro data' severity estimator ■ Properties of 'micro data' severity estimator ■ Comparison with average loss

**Mean ultimate loss (severity):**

- Is one of the most important reported indicator of non-life portfolio performance.
- Is an important input entering reserving, pricing and risk models.

**Micro data:**

- Each loss is reported with some initial value which is further adjusted during the settlement process until the claim is closed at some random time.




**Ultimate loss as an aggregate:**

$$X = X_0 \sum_{j=1}^{\omega} I_j F_j$$

$X_0$ ... Initial value at reporting
$F_j$ ... Adjustment from initial to dev. year $j$
$I_j$ ... Indicator in which dev yr. is the loss closed

**Assumptions:**

- Maximum development year ω is known and deterministic.
- Incremental adjustments are mutually independent.
- Incremental adjustments are independent on closure.
- Initial value is independent on closure and adjustments.
- Finite moments of all variables are assumed.

Note: Indicators $I_j$ are NOT independent; The vector of indicators has multinomial distribution with known correlation structure.

**RESULT 1: Analytic formulas for the expected value and variance of the ultimate loss are under the assumptions derived.**

**Arithmetic average:**

- First choice for the ultimate loss estimator is often the simple arithmetic average.
- Does not use detailed loss data commonly available.
- Alternatively, estimators of the initial loss, loss adjustments and probability of closure are estimated.

**Micro data estimator:**

- For fixed number of losses, closures are randomly spread over development years => Random number of observations for the adjustment factors.
- Censoring not assumed in this work.
- More parameters are estimated but using unaggregated data.
- **IS IT WORTH IT?**

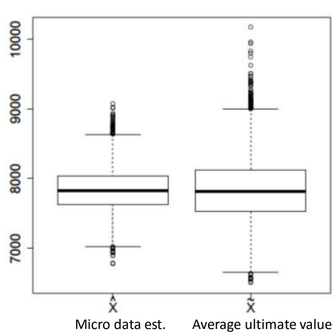
**RESULT 2:**

- **Analytic formula for the expected value of the micro data estimator is derived.**
- **The estimator is asymptotically unbiased.**
- **First order approximation of variance of the micro data estimator is derived.**

$$\hat{X} = \hat{X}_0 \sum_{j=1}^{\omega} \hat{p}_j \hat{\mu}_j$$

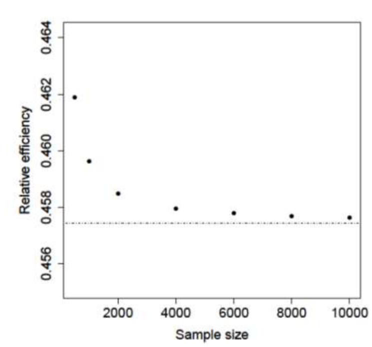
**Simulation study conducted:**

- 5000 losses generated 10 000 times from 'true' values observed in a real MTPL portfolio.
- Maximum ω = 9 development years.
- Gamma distribution for initial loss and adjustments.
- Other 'true values' were also tested.
- Higher efficiency was not proved in general.
- Results show massive gains in efficiency in case of micro data estimator.




**RESULT 3:**

- **Difference in efficiency between the micro data estimator and simple average can be massive.**
- **In our simulation, variability was reduced by almost 55 %.**




**Main references**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