Lapse risk in life insurance: correlation and contagion effects among policyholders' behaviors

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A word on the lapse risk

What is the **lapse risk** ?
→ Insured exiting the portfolio due to :
death, maturity, change of premium level, surrender, ...

Why so much interest ?

1. Among the 3 major risks in life insurance ;
2. Understand the behaviours, and design new products ;
3. Predictions : risk management (ALM).

Lapse classification in life insurance

The 2 main historical explanations for surrenders are ([Out90])

- *liquidity needs* → idiosynchratic → structural surrenders ;
- *economic distress* → environment → temporary surrenders.

Current context : never experienced such low interest rates ⇒ impact on the underwriting of new business...

Threat : massive (temporary) surrenders due to ↗ of interest rate.

Estimation of structural lapses

- **GLM** models for segmentation

- **QIS 5 (EIOPA)** : The lapse rate (LR) is based on the max b/w (1) and (2) :

Step (1) : shocks applied to structural LR (misestimation).
$LR_{up} = \min(100\%, 150\% \times LR) \rightarrow$ our context !
$LR_{down} = \min(0, \max(50\% \times LR, LR - 20\%))$.

Step (2) : mass lapse event, $\sim$ “bank run”.
30%-loss of the sum of positive surrender strain over portfolio ;

- Internal model and practical approach (**S-shaped**)

Still some issues to deal with

+ Pros :
    - easy-to-understand, easy-to-implement,
    - integrates (artificially) copycat behaviours → correlation risk ;

- Cons :
    - not fully realistic ;
    - this is a static model...(does not depend on time t),
    - does not consider the contagion between policyholders...

⇒ We'd like to introduce a model that copes with both correlation and contagion risks to define extreme scenarios.

Dynamic contagion process for the lapse intensity

→ Extended Hawkes, [DZ11].
→ $(N_t)_{t\geq 0}$: counting process of lapses over the whole portfolio.

$$\lambda_t \;=\; \lambda_\infty + (\lambda_0 - \lambda_\infty) e^{-\beta t} + \sum_{i\geq 1} X_i \, e^{-\beta(t-T_i)} 1_{T_i \leq t} + \sum_{j\geq 1} Y_j \, e^{-\beta(t-\hat{T}_j)} 1_{\hat{T}_j \leq t} \tag{1}$$

where lapses occur at T_i, and X_i, Y_j are magnitudes of jumps ($\sim$ Exp. with param. γ and δ).

1. Structural surrender forces λ_0, and λ_∞ (constant here),
2. Temporary surrenders, with
   - endogenous shocks : **contagion**, internal ;
   - exogenous shocks : history of $\hat{N}_t$ → dynamic dependence, source of **correlation** in our setting (to be defined later).

Cause of correlation : interest rate movements

$\rightarrow$ Consider a contract with

- **credited rate** R_t^c : minimum profitability + potential profit benefit.

$\rightarrow$ Let $(r_t)_{t \geq 0}$ be the interest rate with GBM dynamics (μ, σ).

Q : in critical scenarios, how the surrender decision could be affected by the level of r_t ?

Look at the following standardized spread :

$$RG_t^0 := \frac{r_t - R_0^c}{R_0^c}$$

→ Makes sense to ↗ the propensity to lapse when RG_t^0 ↗ ;

→ Say that policyholders would exercise their option to surrender at time $\hat{T}_1$ being the first time RG_t^0 hits a constant barrier $B > 0$.

→ Assume that the company can then adjust the credited rate R_t^c depending on the interest rate level (to avoid massive lapses).

This defines the new standardized spread RG_t^1, given by

$$RG_t^1 = \frac{r_t - R^c_{\hat{T}_1}}{R^c_{\hat{T}_1}} = \frac{r_t - r_{\hat{T}_1}}{r_{\hat{T}_1}}, \qquad \hat{T}_1 \leq t < \infty.$$

Next adjustment will be operated as soon as $RG_t^1 = B$, and so on...

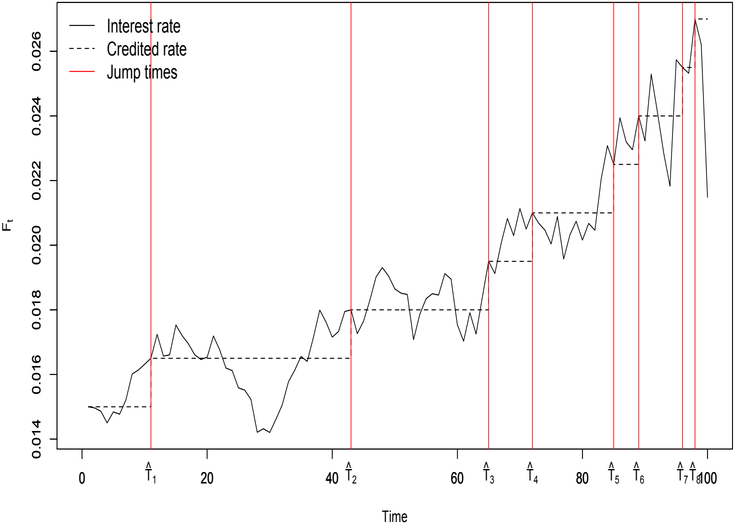
$\Rightarrow$ **These events thus characterize the sequence** $(\hat{T}_j)_{j=0,1,\dots}$ s.t.

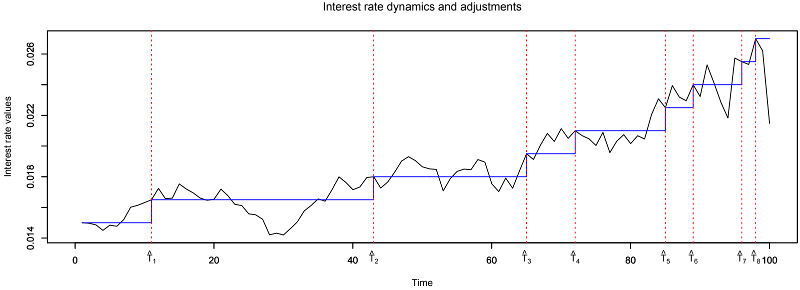
$$\hat{T}_{j+1} \quad = \quad \inf\{t > 0,\ RG_t^j = B\}, \tag{2}$$

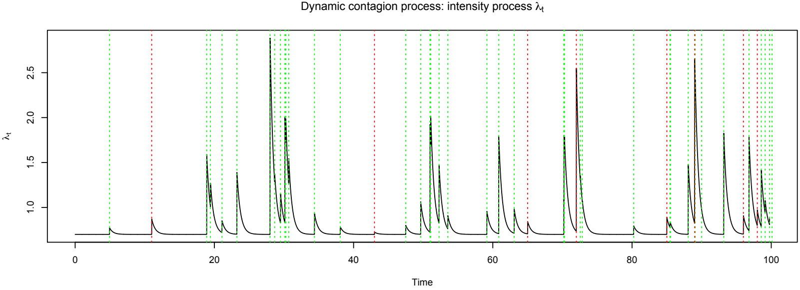
with $\hat{T}_0 = 0$ for convenience.

($\hat{N}_t = \sum_{j\geq 1} 1_{\hat{T}_j \leq t}$: counting process associated to such events.)

Adjustments of the credited rate




Interest rate dynamics and adjustments
Interest rate values
0.014
0.018
0.022
0.026
0
20
40
60
80
100
$\hat{T}_1$
$\hat{T}_2$
$\hat{T}_3$
$\hat{T}_4$
$\hat{T}_5$
$\hat{T}_6$
$\hat{T}_7$
$\hat{T}_8$
Time


Dynamic contagion process: intensity process λ_t
λ_t
1.0
1.5
2.0
2.5
0
20
40
60
80
100
Time

About external jumps at $\hat{T}_j$

→ $(r_t)_{t \geq 0}$ follows a GBM ($\log(r_t/r_0) = (\mu - \sigma^2/2)t + \sigma W_t$).
→ $(\hat{T}_j)_{j=0,1,\ldots}$ are hitting times of the process RG_t.

⇒ The events $\hat{T}_j$ **can also be characterized as follows**

$$\hat{T}_j \quad = \quad \hat{T}_{j-1} \; + \; \inf\{t \geq 0, \quad \mu t + \sigma W_t = \log(1+B)\}.$$

⇒ Inter-arrival times $\Delta \hat{T}_j = \hat{T}_j - \hat{T}_{j-1}$ are i.i.d., with distribution F (inverse Gaussian, [SK91]) :

$$\Delta \hat{T}_j \sim \mathcal{IG}(\theta_1, \theta_2),$$

with $\theta_1 = 2\log(1+B)/(2\mu - \sigma^2)$, $\theta_2 = \log(1+B)^2/\sigma^2$.

→ $\lambda_t, (\lambda_t, N_t), (\lambda_t, N_t, \widehat{N}_t)$ not Markovian.

→ $\hat{T}_j = \sum_{k=1}^{j} \Delta \hat{T}_k$.

Introduce $P(\hat{T}_j \leq t) = F^{j*}(t)$.

→ Denote by $h(t) = E[\hat{N}_t] = \sum_{j=0}^{\infty} P(\hat{N}_t \geq j)$, thus

$$h(t) = \sum_{j=0}^{\infty} F^{j*}(t). \qquad (3)$$

The CDF F^{j*} is still $\mathcal{IG}$, **with mean** $j\theta_1$ **and shape** $j\theta_2$.

Moments of the lapse intensity

→ Assume $\beta\gamma > 1$ (stationary condition).
→ Let

$$m(t,\theta) = E[e^{\theta\lambda_t}],$$

and $m^{(n)}(t,\theta)$: n^{th} derivative of m with respect to θ.
We have

$$m^{(n)}(t,0) = E[\lambda_t^n].$$

→ Denote respectively $\xi(t,\theta)$ and $\widehat{\xi}(t,\theta)$ the m.g.f. of

$$Z_t = \sum_{i=1}^{N_t} X_i e^{\beta T_i} \quad \text{and} \quad \widehat{Z}_t = \sum_{j=1}^{\widehat{N}_t} Y_j e^{\beta \widehat{T}_j}. \tag{4}$$

$Z_t, \widehat{Z}_t$ are discounted compound renewal processes ([LGFW10]).

Similarly, $\xi^{(n)}(t,\theta)$ and $\widehat{\xi}^{(n)}(t,\theta)$ refer to the n^{th} derivative $\xi(t,\theta)$ and $\widehat{\xi}(t,\theta)$ with respect to θ.

→ λ_t can be written in the following form

$$\lambda_t = (\lambda_\infty + (\lambda_0 - \lambda_\infty)e^{-\beta t}) + e^{-\beta t} Z_t + e^{-\beta t}\widehat{Z}_t.$$

⇒ We can then derive

1. the m.g.f. of Z_t and $\widehat{Z}_t$;
2. the m.g.f. of λ_t in function of those of Z_t and $\widehat{Z}_t$;

⇒ At the end, we obtain a recursive formula.

(1) Moment generating functions of Z_t and $\widehat{Z}_t$

The m.g.f. ξ and $\widehat{\xi}$ of Z_t and $\widehat{Z}_t$ are given by **recursive formulas** :

$$
\begin{aligned}
E[e^{\theta Z_t}] &= \xi(t,\theta) = 1 + \int_0^t \left(\frac{\theta e^{\beta u}}{\gamma - \theta e^{\beta u}}\right) \xi(t-u, \theta e^{\beta u}) m^{(1)}(u,0) du, \\
E[e^{\theta \widehat{Z}_t}] &= \widehat{\xi}(t,\theta) = 1 + \int_0^t \left(\frac{\theta e^{\beta u}}{\delta - \theta e^{\beta u}}\right) \widehat{\xi}(t-u, \theta e^{\beta u}) dh(u),
\end{aligned}
$$

→ We can derive the moments of the renewal processes ;
→ The first moment of the intensity λ_t is key (self-excited) ;
→ Recall that $h(t) = E[\hat{N}_t] = \sum_{j=0}^{\infty} F^{j*}(t)$.

(2) Moment generating function of λ_t

Proposition. For $n > 1$, the n^{th} derivative of the surrender intensity m.g.f. is given recursively :

$$
\begin{aligned}
m^{(n)}(t,\theta) &= K(t,\lambda_0,\lambda_\infty)\, m^{(n-1)}(t,\theta) \\
&\quad + \sum_{i=0}^{n-1} G(i,n)\Big(I_i(t,\theta) + \widehat{I_i}(t,\theta)\Big)\, m^{(i)}(t,\theta),
\end{aligned}
\tag{5}
$$

with I_k and J_k for $\{k = 1, 2, \ldots\}$ given by

$$
\begin{aligned}
I_k(t,\theta) &= I_{k-1}^{(1)}(t,\theta) + H(I_{k-1}(t,\theta))\, \xi^{(1)}(t,\theta e^{-\beta t}), \\
\widehat{I_k}(t,\theta) &= \widehat{I}_{k-1}^{(1)}(t,\theta) + H^{'}(\widehat{I}_{k-1}(t,\theta))\, \widehat{\xi}^{(1)}(t,\theta e^{-\beta t}).
\end{aligned}
$$

Application : expected intensity process

$\rightarrow$ The expectation $E[\lambda_t]$ is given by

$$m^{(1)}(t,0) = \left(\lambda_0 - \frac{\beta\lambda_\infty}{\beta - 1/\gamma}\right) e^{-(\beta - \frac{1}{\gamma})t} + \frac{\beta\lambda_\infty}{\beta - 1/\gamma} + \frac{1}{\delta}\int_0^t e^{-(\beta - \frac{1}{\gamma})(t-s)} h'(s)ds.$$

Remark : $m^{(1)}(t,0)$ comprises an infinite series associated with the external jumps component ($h'(s) = \sum_{j=0}^{\infty} f^{j*}(s)$).

Trick to get closed-form expressions : convolution of exponential and inverse gaussian r.v.

$\rightarrow E[N_t] = E\left[\int_0^t \lambda_s ds\right] = \int_0^t m^{(1)}(s,0)ds.$

Limiting behaviour of the lapse intensity

We can also compute the limit of this expectation :

$$\lim_{t\to\infty} E[\lambda_t] = \frac{\beta\lambda_\infty}{\beta - 1/\gamma} + \frac{1}{\delta\theta_1(\beta - 1/\gamma)}. \tag{6}$$

→ The limiting behavior of the lapse intensity first moment strongly depends on the limit of the last term in the previous result.

→ Serfozo [2009] for such results.

Application to risk management - calibration

Some parameters can be calibrated from practitioners' knowledge :

- λ_0 (initial force of lapse) is a constant.
  $\Rightarrow$ Exponential underlying lifetime distribution before lapse.
- λ_∞ can be fixed by the risk managers as their goal...
  $\Rightarrow$ When the time horizon is given, this can be easily fixed.
- B depicts the sensitivity of PH to opportunities (experts).

Some parameters (e.g. GBM) should be calibrated from empirical data / history whenever possible.
Others relate to the management : β (ability to reassure the PH), γ, δ tie in with the mean size (SI) of lapsed contracts...

Stress tests : comparison with SII and S-shaped

→ **Within the Solvency II framework** : run-off, 1-year horizon.
→ **With regard to financial context** : focus on the upper-shock.
→ Risk measures under consideration : *VaR* and *TVaR*.

Parameters		**Solvency II Standard formula** Risk level	Shocks	**S-shaped curve (ONC)** Risk level	Shocks	**Hawkes counting process** $E[N_t]$	VaR_α	$TVaR_\alpha$	**Dynamic contagion process** $E[N_t]$	VaR_α	$TVaR_\alpha$
	10%								455	1028	1142
B	30%	75	112	75	375	291	776	837	312	818	930
	50%								293	778	886
	0.1								2461	4286	4559
δ	0.5	75	112	75	375	291	776	837	702	1460	1594
	1.5								455	1028	1142

TABLE – Impact of contagion and correlation on $VaR_\alpha(N_t)$, $TVaR_\alpha(N_t)$ at level $\alpha = 99.5\%$, in a 1-year time horizon ($t = 250$).

Conclusion on stress tests

→ The shock in SII looks neither consistent nor realistic.
→ Stress tests in most of companies seem to be **underestimated**.

→ OK for extreme scenarios (reserving), not so realistic in classical regime (pricing).

→ Sensitivity to IR movements is obviously not linear...

→ External component has a limited impact, provided that mean size of the external jumps is low ⇒ portfolio composition is crucial !

Perspectives :

1. Calibration on a real-life portfolio ;
2. Retrieve the whole distribution of lapses N_t,
3. Extend this approach with an adapted interest rate model.

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