A stochastic simulator model for the Limit Order Book

10 août 2016

Focus

- Modern financial markets use a continuous double auction mechanism to store and match orders and facilitate intraday trading.
- advent in computer power, latency, data mining allow to increase the speed of financial institutions (banks, hedge funds, brokers, market makers), to receive informations, treat it, and send orders to buy or sells.
- Brokers and Market Makers manage important volumes of shares to trades everyday. Their trades because of large size have an significant impact on the price called *Market impact* because financial markets participants adjust almost immediately to strong changes in liquidity or prices (Almgren ,Thum and Hauptmann (2005), Engle, Ferstenberg and Rusell(2005), Ferraris (2007), Moro and al.(2009)).
- Most of their activities relies on high- frequency trading algorithms. Example VWAP for a broker.
- Need to backtest strategies. Mostly two approaches to tackle this issue :
  - replay it on past datas : but here the trade of the strategy are *virtuals*.
  - check the Profit and Loss on real-time trades and adapt the strategy...

Aims of the presentation

- Presenting an agent-based model simulator able to dialog with a HFT algorithm (Rama Cont and al. (2010)).These agents are simples, called Zero Intelligence, they rely on a simple small set of parameters. However, their simple rules of decisions leads to a macroscopic behaviour of the market able to replicate interesting stylized facts observed on the market.
- assessing the two main properties of this model :
  - stock prices follow a brownian motion on the long run
  - able to capture realistically the persistence effect on prices on the long run of shocks of liquidity. Allowing to evaluate the market impact of a strategy.
- calibration procedure of the parameters that lead their decisions, allowing to capture many observed stylized facts, related to : Bid-Ask spread, volatility of prices, available liquidity around the mid price.
- Application of this model to evaluate on real datas, the market impact of :
  - a TWAP algorithm
  - 3 market making strategies (Ait-Sahalia and Saglam (2010))

Plan of the Presentation

1. Stylized facts of the LOB
2. The Stochastic Simulator
3. Backtest of HFT strategies

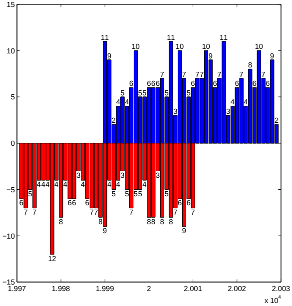
The Limit Order Book

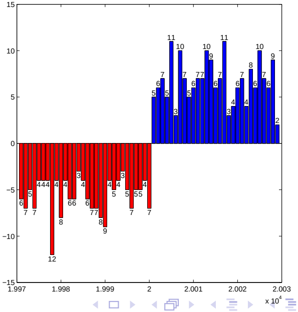
- Mismatch between buyers and sellers at any given instants solved via an order based market with two kinds of basic orders.
  - Impatient traders submit *Market orders* : buy/sell at all price a given quantity.
  - more patient ones submit *Limit order* : Orders to buy/sell a given quantity with a limit price.
- LOB : discrete price grid populated by orders of two species (buy or sell) of variable size (multiple of the BOARDLOT = 100 shares).
- the coordinates of the order book are the limit prices sorted by ascending number of prices incremented by the tick size of the market (1 cent).
- Buy order go to the **Bid** side, while sell orders to the **Ask** side

The State of the Market

- Between 9 am and 9 :30, all markets participants presents their bid and offers by limit orders. At 9 :30, the two curves of bid and demand are cleared by a mechanism called *fixing*, where all trades occur at the same price, called the *fixing* price.
- after 9 :30am : non matched remaining orders lies in the LOB and the Continuous Double Auction Period begins.
- at 5pm : similar period of Call Auction, leading to the last price of the day, called the *closing price*.

- Definition
  - Best Ask price : a_t
  - Best Bid price : b_t
  - such that $b_t < a_t$ during the entire CDA Period.





Stylized facts of the LOB on real datas : The datas

- We used events data on three tickers ABX, BBD and RY observed on TSX trading venue.
- With such detailed datas it is possible to rebuild the book update after update.

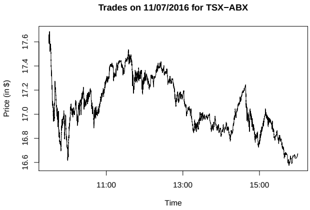
In particular, we can observe

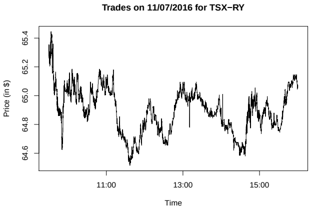
1. spread at any update of the book.
2. traded volumes at any instant
3. compute volatility
4. number of limit orders arrival at any distance from their opposite best prices
5. number of cancellations of limit orders occuring at any distance from their opposite best prices
6. a curve of profile of volumes, is number of shares available at any given distance from the opposite best price.
7. a curve of profile of number of orders, which is the number of orders available at any given distance from the opposite best price.
8. collect all the limit orders arrived in the market and obtain the distributions of their size.

These statistics will be crucial to calibrate the parameter of our model. We present them before going to the presentation of the model.

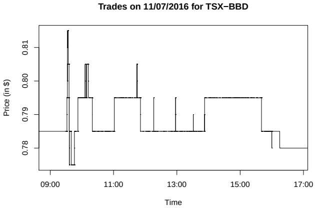
Stylized facts of the LOB on real datas : The datas

We focus our analysis on 3 financial tickers : ABX, BBD and RY observed on TSX trading venue the 11/02/2016.

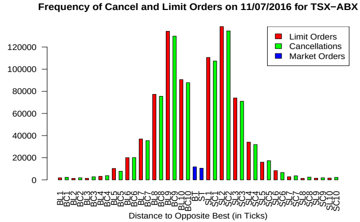


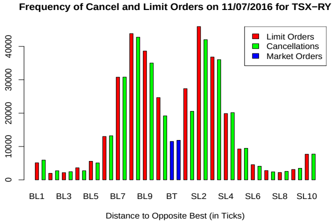


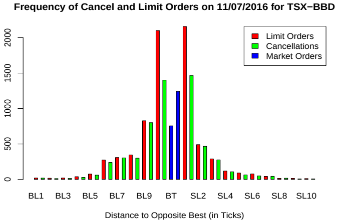
Ticker	ABX	RY	BBD
1min vol (ann.)	90.8%	22.5%	81%
TSRV (tick/h)	35.504	33.096	1.771
BCRV (tick/h)	39.006	31.183	1.778
MSRV (tick/h)	35.422	32.621	1.726
RV (tick/h)	32.844	30.990	1.842
Traded shares	9530807	4158593	5115608
Spread (Average)	1.264	1.707	1.044
Spread (std)	0.48	2.38	0.20
Spread (Median)	1	2	1
Spread (Q75)	1	2	1



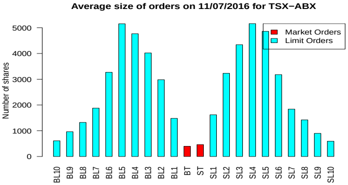
Stylized facts of the LOB on real datas : Arrival rates of Orders

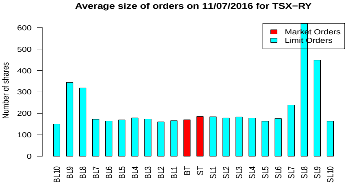






Stylized facts of the LOB on real datas : Average size of Orders





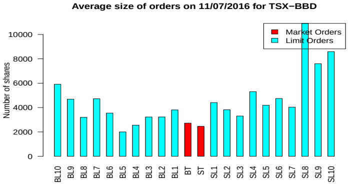


Figure – Average Size of limit market orders and cancellations oberved on 11/02/2016 on TSX market for ABX, RY and BBD tickers

Stylized facts of the LOB on real datas : Average Volume present in the LOB

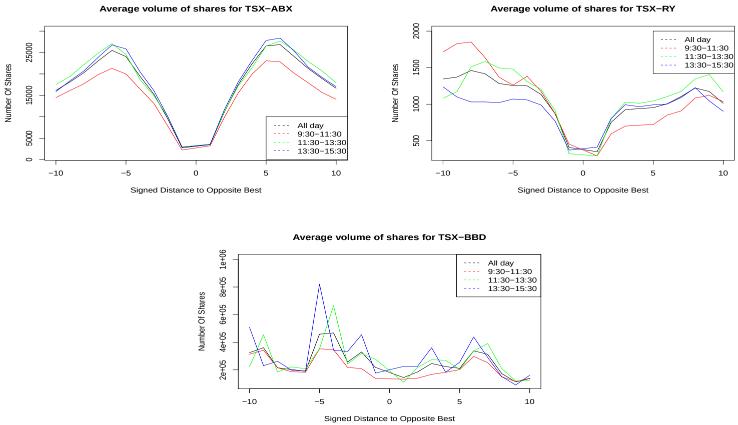


Figure – Average Volume of limit orders available in the LOB oberved on 11/02/2016 on TSX market for ABX, RY and BBD tickers

Stylized facts of the LOB on real datas : Average Number of Orders present in the LOB

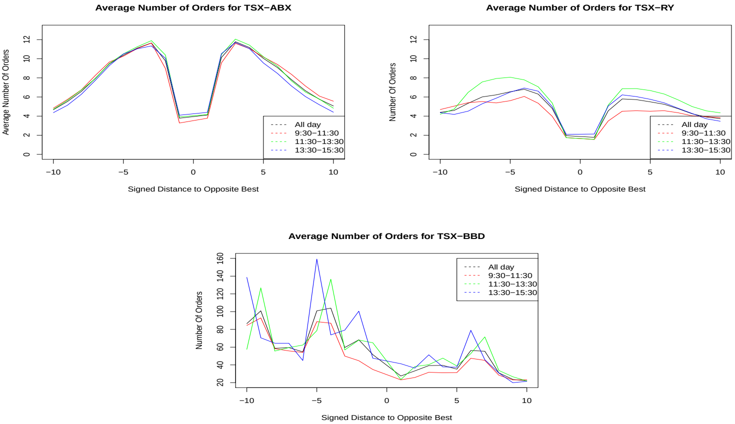


Figure – Average Number of limit orders available in the LOB oberved on 11/02/2016 on TSX market for ABX, RY and BBD tickers

The Zero Intelligence Model

In the ZI framework (Cont, Stoikov, Talreja (2010) and Abergel, Jeddidi (2013)), an initial LOB (say at time t=9 :30), is impacted in three possibles ways

Trades : by Buy/Sell markets orders of size ξ^M arriving according to Poisson(λ).

Depositions : By Buy/Sell limit orders of size ξ_i^L that arrive at a distance of i ticks from the opposite best quote according to a Poisson process with rate $\alpha(i)$, with $1 \leq i \leq B$.

Cancellations : any outstanding order present in a LOB either at Ask side either at Bid side and located at a distance of i ticks ($i \leq B$) from the opposite best quote has a remaning life-time drawn from $\mathcal{E}\mathrm{xp}(\delta_i)$.

Set of parameter : $(\lambda, \xi^M, \alpha_{\mathbf{1}}, \ldots, \alpha_B, \delta_{\mathbf{1}}, \ldots, \delta_B, \xi_{\mathbf{1}}^L, \ldots, \xi_B^L)$, if the ξ_i^L's are assumed constants.

In our simulations, we will take the size of limit orders ξ_i^L, random. Extensive Simulations show indeed that constant size limit order size does not allow a good fit of the volatility.

We will fit to the size of orders two types of distributions :

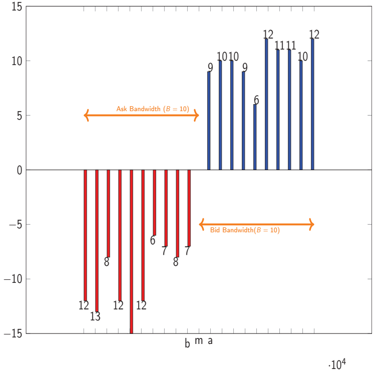
Log normal : $\log(\xi_i^L) \sim \mathcal{N}(\mu_i, \sigma_i)$

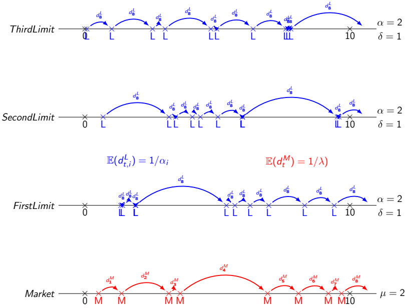
Johnson : $\xi_i^L \sim Johnson(\mu_i, \zeta_i, \kappa_i, \tau_i)$

The Johnson distribution is a class of family of 4 parameters that includes the Log Normal distribution, the Gamma and the Exponential distribution, so makes the fit to order size more robust accross different assets.

Simulation of the ZI Model in practice

- 2B "Zero Intelligence" agents providing liquidity to Sell/Buy : the i-th Buyer/Seller agent providing liquidity at $(a_t - i\tau)/\ (b_t + i\tau)$ ($\tau=$ tick size).
- 2 agents consuming the liquidity : one seller, one buyer at all prices, with interarrival times of their orders following $\mathcal{E}xp(\lambda)$.
- their behaviour tend to fill the spread, which ensures stationarity of the spread and equilibrium.
- dynamic of the price is *endogenous*.
- the simulation run from 9 :30 to 17 :00 by discrete events.





A simplified description to simulate on $[0, T]$ is Abergel and Jedidi (2010) :

1. Initialization : set $t = 0$ and define $A = 2\sum_{i=1}^{B} \alpha(i)$,
2. Compute for all $1 \leq i \leq B$ $Na(t, i)$ the number of oustanding shares in the Ask Side present at i ticks from b_t, and $Nb(t, i)$ the number of oustanding shares in the Bid Side present at i ticks from a_t. Then compute
$$Na^*(t) = \sum_{i=1}^{B} \delta_i Na(t, i) \quad \text{and} \quad Nb^*(t) = \sum_{i=1}^{B} \delta_i Nb(t, i)$$
3. Compute $S = 2\lambda + A + Na^*(t) + Nb^*(t)$ and draw an event $e = 1, 2, 3, 4$ in the relative probabilities $\{2\lambda, A, Na^*(t), Nb^*(t)\}$.
4.
   - If $e = 1$, generate a market order and draw its signs.
   - If $e = 2$, we have to generate a limit order. Generate its sign and draw an event $e^* \in \{1, \ldots, B\}$ in the relative probability $\{\alpha(1), \ldots, \alpha(B)\}$. If its sign $\epsilon = -1$, its limit price is $a_t - e^*\gamma$ and if $\epsilon = +1$, its limit price is $b_t + e^*\gamma$.
   - If $e = 3$, generate a cancellation at the ask side. To choose the order, draw an event $e^* \in \{1, \ldots, B\}$ in the relative probabilities
   $$\{\delta_1 Na(t, 1), \ldots, \delta_B Na(t, B)\}$$
   . Then cancel one order among those present at the limit price $b_t + e^*\gamma$.
   - If $e = 4$, generate it at the bid side. To choose the order, draw an event $e^* \in \{1, \ldots, B\}$ in the relative probabilities
   $$\{\delta_1 Nb(t, 1), \ldots, \delta_B Nb(t, B)\}$$
   . Then cancel one order among those present at the limit price $a_t - e^*\gamma$.
5. Update the LOB. Generate $\tau \sim \mathcal{E}xp(S)$, and update the time $t = t + \tau$.
6. Repeat steps 2-5 until $t > T$ stopping time.

The model can dialog with any trading algorithm, that can place an order at any given instant, allowing to obtain real impact of its orders on the future dynamic of the LOB.

Simulation of the ZI Model in practice

1. Guo (2005) present an example of backtest of TWAP.
2. backtest of market-making strategy Cont and al.(2010)
3. Toth Lemperiere and al. (2011) compare the real market impact of some metaorders to those obtained from this model. It is able to replicate for a metaorder of size Q placed on a day where the volume exchanged is V and volatility σ the following shape of market impact

$$\Delta(Q) = Y\sigma\sqrt{\frac{Q}{V}}$$

Calibration of the ZI Model

We followed the procedure presented in Abergel and Jedidi (2010) to calibrate

$(\lambda, \xi^M, \alpha_1, \ldots, \alpha_B, \delta_1, \ldots, \delta_B)$ on event datas gathered on a period of time of length T.

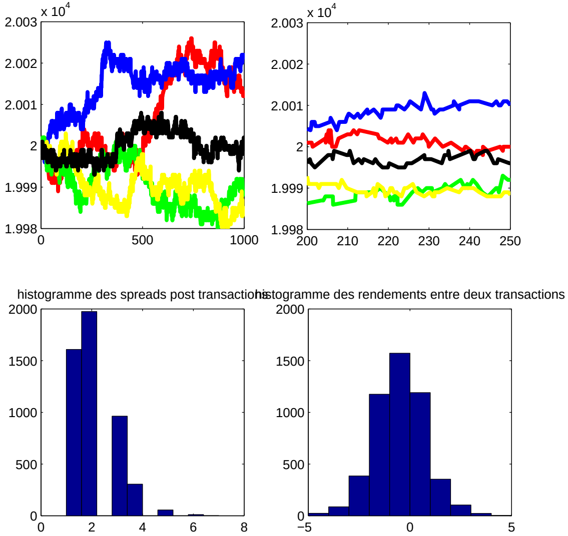
- $\lambda = \frac{\#\text{Trades}}{2T}$
- ξ^M = Average Trade Size
- $\alpha_i = \frac{\#\text{Limit Orders arrived at a distance i from the Opposite Best Price}}{2T}$
- Compute V_i the average volumes observed at a distance i from the opposite best price.

$$\delta_i = \frac{\#\text{Cancellations occuring at a distance i from the Opposite Best}}{2V_i T}$$

This way, the simulation in a given period one time posesses

- same number of trades that those observed on the market, with same size, so in average same traded volumes
- same number of arrival of limit orders as those observed on the market for each distance of the opposite best.
- the calibration of δ_i, managing the cancellations is the less intuitive part of the calibration. It aims to maintain an average profile of volume of orders similar to the one observed on data.
- Intuition : if the profile volume is well fitted, when a trade hits the book, the deviation of prices in average should be the same as the observed one, leading to a good volatility and a good spread.

Returns



x 10^4
2.003
2.002
2.001
2
1.999
1.998
0
500
1000
x 10^4
2.003
2.002
2.001
2
1.999
1.998
200
210
220
230
240
250
histogramme des spreads post transactions
2000
1500
1000
500
0
0
2
4
6
8
histogramme des rendements entre deux transactions
2000
1500
1000
500
0
−5
0
5

Comparison between simulations and real datas : Average Volume present in the LOB

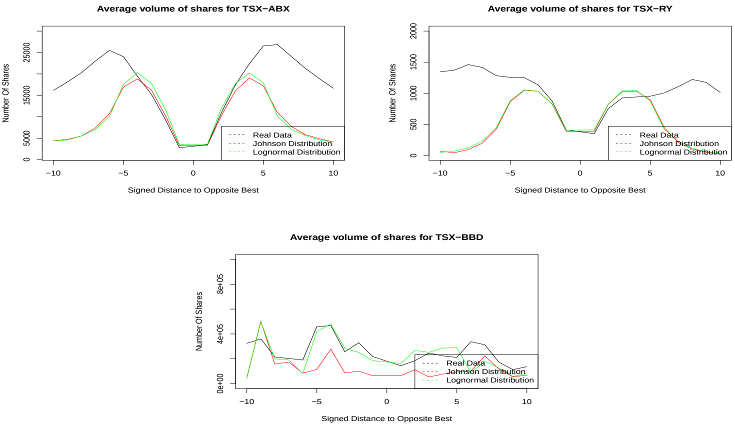


Figure – Average Volume of limit orders available in the LOB oberved on 11/02/2016 and simulated on TSX market for ABX, RY and BBD tickers

Comparison between simulations and real datas : Average Number of Orders present in the LOB

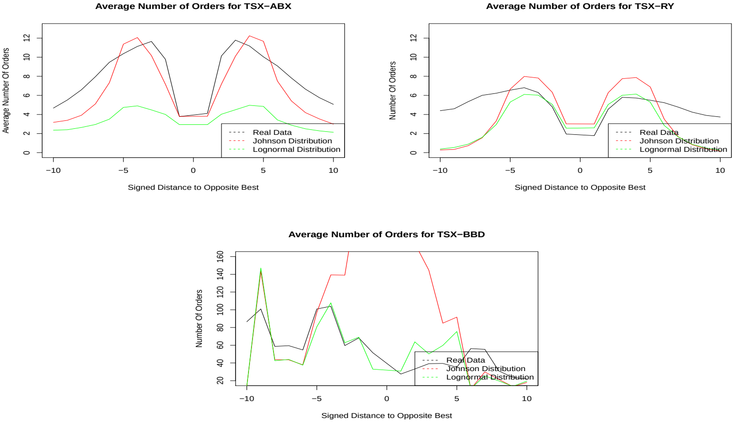


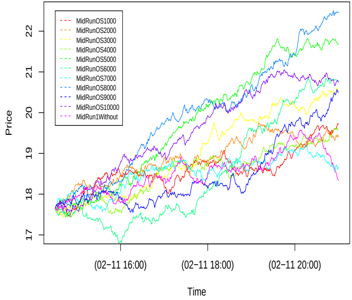
Figure – Average Number of limit orders available in the LOB oberved on 11/02/2016 and simulated on TSX market for ABX, RY and BBD tickers

Comparison between simulations and real datas : Volatility and Spread

Ticker	ABX			RY			BBD		
Mode sim.	Real	LogN	John.	Real	LogN	John.	Real	LogN	John.
1min vol (ann.)	90.8%	89.8%	52.1%	22.5%	11.6 %	9.1%	81%	0%	0%
TSRV (t/h)	35.5	38.6	21.1	33.1	16.7	12.85	1.8	0	0
BCRV (t/h)	39	38.1	21.1	31.1	14.5	11.6	1.8	0	0
MSRV (t/h)	35.4	39	21.1	32.6	16.7	12.85	1.7	0	0
RV (tick/h)	32.8	39.7	22.6	31	16.5	12.15	1.8	0	0
Spread Av.	1.26	1.1	1.03	1.7	1.1	1.09	1.05	1	1
Spread std	0.48	0.3	0.17	2.38	0.35	0.28	0.20	0	0
Spread Med	1	1	1	2	1	1	1	1	1
Spread Q75	1	1	1	2	1	1	1	1	1

TWAP market impact

To illustrate the ability of the model to simulate market impact, we run 10 algos. Every 1 minutes, we place market orders to buy 1000, 2000, ...,10000 shares with the fitted LogNormal Model on TSX-ABX, every minute across the day.

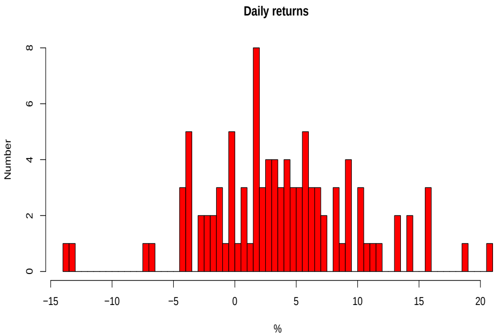


Three conclusions :

1. the simulated prices gets a much stronger upward trend for very large value of the market order size
2. this conclusion is not the same for the small values.
3. we need to proceed to monte carlo and run many trajectories to detect the market impact in the case say 1000 shares.

TWAP market impact

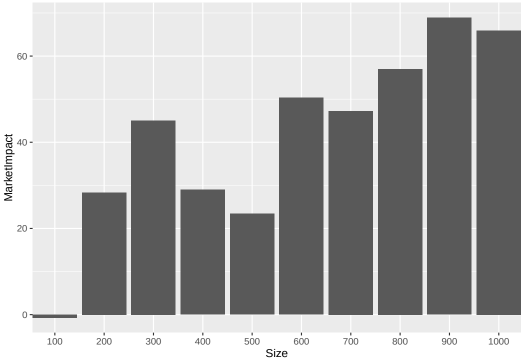
We run 100 trajectories to detect the market impact in the case 1000 shares.



Market Impact : $E(p_T - p_{\mathbf{0}})/0.01 = 65.91$ ticks

TWAP market impact

We do the same simulation exercise for 100, 200, . . . 1000 shares every minutes.



Conclusion :

- Icreasing Market impact. No market impact for 100 shares.
- should increase the number of Runs. 100 is not sufficient.

Market Making Strategies

We compare 3 market making strategies :

Naive Agent : this agent bid and ask at the top of book all the times. Every times he is executed at one side, he post a new order at this side. If one of the two best price moves, he amends his order to stay at the top of book

Fixed Inventory Agent : similar to the Naive Agent, but has fixed limit inventory. When one inventory limit is reached at Buy/Sell, he will stop posting Buy/Sell orders, until this limit constraint is relaxed

Optimal Agent : designed by Ait-Sahalia and Saglam (2010). It is a fixed inventory Agent with the main difference that it receives signals about the sign of the next incoming market order.

Input of the Agents : All these agents subcribe to the Book updates of the LOB.

We will account for delays in the transfer of information from the agents to the Trading Venue :

Incoming Book Updates delay : time that it takes for a message containing a Book Snapshot to arrive to the Agents. Reasonnable estimation : 1ms.

Posting delay : It is the delay for the sent order to actually be matched in the LOB. Reasonnable estimation : 1ms.

Ack delay : After a posted order has been traded, or after the order is matched to the book, a message of acknowledgment is sent by the Trading Venue. Any algorithm should wait for this "Ack" before posting any new order. Reasonnable estimation : 1ms.

These delays impact the P&L of any HFT market making strategy and they have to be accounted for in the simulation. More over, their effect on the P&L is tricky to evaluate. Only real backtest can help to evaluate.

Market Making Strategies Decisions

At any given instant t, the Optimal Agent of Ait-Sahalia and Saglam (2010) receives a signal s_t telling him if the next incoming order will be a Buy (B) or Sell (S).

- Definінf its policy to quote at the bid $\ell^{b*}(x, s_t)$ and quote at the ask $\ell^{a*}(x, s_t)$, which
- Its quoting policies are determined this way.

$$\begin{aligned} \ell^{b*}(x, S) &= \mathbb{1}_{x<U^*}, \quad \ell^{a*}(x, B) = \mathbb{1}_{x>-U^*} \\ \ell^{a*}(x, S) &= \mathbb{1}_{x>L^*}, \quad \ell^{b*}(x, B) = \mathbb{1}_{x<-L^*} \end{aligned}$$

  where x is the current inventory and $L^* < 0 < U^*$
- U^* is the limit for the Agent to capture the signal. (Sell if s=B and Buy if s=S).
- L^* is the limit for the Agent to place an order in the opposite direction of the signal. (Sell if s=S and Buy if s=B).
- The optimal agent will always quote to buy when his inventory is negative and sell when his inventory is positive. The limit are there to bind the inventory in the direction of the signal.
- $U^*$and L^* are determined by the value of average spread, frequency of trades, volatility, average size of jump in prices and the probability p of s_t to give a good prediction. Ait-Sahalia and Saglam (2010) provides algorithms to determine them and proves that these policies are optimals for a Dynmical Programming problem where the prices are random walk and the spread is constant.
- the higher is p, the more different U^* and $-L^*$ will be.
- The Fixed Inventory Agent consists just in assuming $p = 0.5$ and thus $U^* = -L^*$.

Market Making Strategies P&L sensibility

P&L of such market making strategies are sensible to

Latency Effects : Tricky to evaluate.

Queuing priority : if all posted orders are always last in the queue of a given level, it may never get executed, or its execution will be followed by a move in price in its direction. Market making strategies exploits the bid ask bounce of prices and are profitable when the orders posted at Buy and Sell are executed one after the other one.

Volatility and spread : if the price move too much or the spread is too low, the gain can decrease dramatically. In particular the Inventory limits can be attained too frequently, which should not happened too frequently.

trend : if the price trend upward or downward, the P&L decreases.

Liquidity in higher levels : if my orders are located at levels with highest levels full of liquidity, then, it is more likely that the price with bouncing around the current bid-ask spread, and they will get both executed.

HFT traders always keep amending, cancelling orders that they place in order to be well located in the queue, at queues where they can take profit from the Wall of liquidity of highest levels. That explains the high level of posting of limit orders and their cancellations.

We run an historical backtest on the 3 tickers with $Delay = 1ms$. With $CX2$ Trading Venue as a signal.

agent	ticker	buys	sells	inventory	pnl
FIAgentWaitAck	ABX	7020	6958	6200	($5,642.00)
NaiveAgentWaitAck	ABX	7209	6956	25300	($11,158.00)
OptimalAgentWaitAck	ABX	7020	6957	6300	($5,703.50)
FIAgentWaitAck	RY	4098	4284	-18600	($3,627.00)
NaiveAgentWaitAck	RY	4097	4527	-43000	($4,494.00)
OptimalAgentWaitAck	RY	4098	4279	-18100	($3,613.50)
FIAgentWaitAck	BBD	64	84	-2000	$21.50
NaiveAgentWaitAck	BBD	64	84	-2000	$21.50
OptimalAgentWaitAck	BBD	61	84	-2300	$20.00

Stochastic Simulator Backtest

We run an backtest of the stochastic simulator on the 3 tickers with Delay = 1ms.

agent	ticker	buys	sells	inventory	pnl	vol
NaiveAgentWaitAck	ABX	8414	8518	-10400	$3362.00	41%
FiAgentWaitAck	ABX	8298	8301	-300	$1986.50	43%
OptimalAgentWaitAck	ABX	8296	8291	500	$1700.50	42%
NaiveAgentWaitAck	RY	4991	5011	-2000	$2651.00	6.2%
FiAgentWaitAck	RY	5051	5113	-6200	$4134.00	6.8%
OptimalAgentWaitAck	RY	5090	5042	4800	$3531.00	7.1%
NaiveAgentWaitAck	BBD	36	37	-100	$36.50	0
FiAgentWaitAck	BBD	37	34	300	$35.50	0
OptimalAgentWaitAck	BBD	50	50	0	$38.00	0

Conclusion

- we provide a complete framework for a stochastic simulator dialoging with trading algorithms that allows to model the market impact .
- behaviour of the model :
  - can be calibrated to obtain proper values of the real volatility, the book liquidity, the number of trades and arrival of limit orders, spread....
  - brownian motion on the long run, with is a desirable feature, with bid ask bounce on the short term horizon.
- the model can be used to assess the market impact on the long run of a TWAP.
- for market making algorithms, need to assess better the queuing priority, because the model is unable to prevent highly active algorithms to "make their own spread".