

Fair evaluation of insurance contracts with performance depending on different investment funds with automatic remixing mechanism

Massimo Costabile, Marcellino Gaudenzi

University of Calabria, University of Udine
Italy

3RD European Actuarial Journal Conference, Lyon, 2016

Motivation

- Competition with financial institutions has induced life insurers to propose increasingly sophisticated policies

Motivation

- Competition with financial institutions has induced life insurers to propose increasingly sophisticated policies
- an example of such a policy is represented by a contract with performance depending on two investment funds with an automatic remixing mechanism

- Competition with financial institutions has induced life insurers to propose increasingly sophisticated policies
- an example of such a policy is represented by a contract with performance depending on two investment funds with an automatic remixing mechanism
- it is offered in the Italian insurance market but we are aware that similar contracts are present also in other European countries

- Competition with financial institutions has induced life insurers to propose increasingly sophisticated policies
- an example of such a policy is represented by a contract with performance depending on two investment funds with an automatic remixing mechanism
- it is offered in the Italian insurance market but we are aware that similar contracts are present also in other European countries
- the policy works as follows: the first investment fund is characterized by low volatility and is less risky than the second one that has a higher volatility

The contract

- Ignoring mortality, a target interest rate, i_g , and a target time horizon, τ , not greater than the contract maturity, T , are fixed

The contract

- Ignoring mortality, a target interest rate, i_g , and a target time horizon, τ , not greater than the contract maturity, T , are fixed
- a single premium, U , is paid by the insured person at the contract inception

The contract

- Ignoring mortality, a target interest rate, i_g , and a target time horizon, τ , not greater than the contract maturity, T , are fixed
- a single premium, U , is paid by the insured person at the contract inception
- a fraction $U(1 + i_g)^{-\tau}$ of the single premium is invested in the low-volatility fund

The contract

- Ignoring mortality, a target interest rate, i_g , and a target time horizon, τ , not greater than the contract maturity, T , are fixed
- a single premium, U , is paid by the insured person at the contract inception
- a fraction $U(1 + i_g)^{-\tau}$ of the single premium is invested in the low-volatility fund
- the remaining part is invested in the high-volatility fund

The contract

- Ignoring mortality, a target interest rate, i_g , and a target time horizon, τ , not greater than the contract maturity, T , are fixed
- a single premium, U , is paid by the insured person at the contract inception
- a fraction $U(1 + i_g)^{-\tau}$ of the single premium is invested in the low-volatility fund
- the remaining part is invested in the high-volatility fund
- at the end of equally spaced time intervals, $t_k, k = 1, \dots, m$, an automatic rebalancing mechanism between the two funds acts as follows

The contract

- Ignoring mortality, a target interest rate, i_g , and a target time horizon, τ , not greater than the contract maturity, T , are fixed
- a single premium, U , is paid by the insured person at the contract inception
- a fraction $U(1 + i_g)^{-\tau}$ of the single premium is invested in the low-volatility fund
- the remaining part is invested in the high-volatility fund
- at the end of equally spaced time intervals, $t_k, k = 1, \dots, m$, an automatic rebalancing mechanism between the two funds acts as follows
- if the low-volatility fund accrues interest at a rate greater than the target rate, the insurer transfers the surplus into the high-volatility fund

The contract

- Vice versa, if the rate of return of the low-volatility fund is lower than the target interest rate, the insurer transfers a sum from the high-volatility fund to the low-volatility fund so that the low-volatility fund value increases at the target interest rate

The contract

- Vice versa, if the rate of return of the low-volatility fund is lower than the target interest rate, the insurer transfers a sum from the high-volatility fund to the low-volatility fund so that the low-volatility fund value increases at the target interest rate
- the target rate is usually set equal to zero from the target date to the contract maturity

The contract

- Vice versa, if the rate of return of the low-volatility fund is lower than the target interest rate, the insurer transfers a sum from the high-volatility fund to the low-volatility fund so that the low-volatility fund value increases at the target interest rate
- the target rate is usually set equal to zero from the target date to the contract maturity
- at maturity the policyholder receives the current value of the two funds

The contract

- Vice versa, if the rate of return of the low-volatility fund is lower than the target interest rate, the insurer transfers a sum from the high-volatility fund to the low-volatility fund so that the low-volatility fund value increases at the target interest rate
- the target rate is usually set equal to zero from the target date to the contract maturity
- at maturity the policyholder receives the current value of the two funds
- a minimum guarantee assuring that the low-investment fund accrues interest at the target rate along the policy lifetime is usually inserted into the contract

The contract

- Vice versa, if the rate of return of the low-volatility fund is lower than the target interest rate, the insurer transfers a sum from the high-volatility fund to the low-volatility fund so that the low-volatility fund value increases at the target interest rate
- the target rate is usually set equal to zero from the target date to the contract maturity
- at maturity the policyholder receives the current value of the two funds
- a minimum guarantee assuring that the low-investment fund accrues interest at the target rate along the policy lifetime is usually inserted into the contract
- at each remix date a fee is applied by the insurer

The contract

- The rationale of the contract is to include in the same policy two different levels of risk. An investment with small risk that aims at preserving the single premium and, at the same time, an investment with an higher risk to capture the opportunity of greater returns furnished by more volatile markets

The contract

- The rationale of the contract is to include in the same policy two different levels of risk. An investment with small risk that aims at preserving the single premium and, at the same time, an investment with an higher risk to capture the opportunity of greater returns furnished by more volatile markets
- for illustrative purposes, we consider a policy with maturity $T = 10$ years, target date $\tau = 5$ years, and target interest rate of 3.5% per annum

The contract

- The rationale of the contract is to include in the same policy two different levels of risk. An investment with small risk that aims at preserving the single premium and, at the same time, an investment with an higher risk to capture the opportunity of greater returns furnished by more volatile markets
- for illustrative purposes, we consider a policy with maturity $T = 10$ years, target date $\tau = 5$ years, and target interest rate of 3.5% per annum
- a single premium, $U = 100$, is paid at the contract inception

The contract

- The rationale of the contract is to include in the same policy two different levels of risk. An investment with small risk that aims at preserving the single premium and, at the same time, an investment with an higher risk to capture the opportunity of greater returns furnished by more volatile markets
- for illustrative purposes, we consider a policy with maturity $T = 10$ years, target date $\tau = 5$ years, and target interest rate of 3.5% per annum
- a single premium, $U = 100$, is paid at the contract inception
- the automatic rebalancing mechanism acts at each anniversary of the contract

The contract

- The rationale of the contract is to include in the same policy two different levels of risk. An investment with small risk that aims at preserving the single premium and, at the same time, an investment with an higher risk to capture the opportunity of greater returns furnished by more volatile markets
- for illustrative purposes, we consider a policy with maturity $T = 10$ years, target date $\tau = 5$ years, and target interest rate of 3.5% per annum
- a single premium, $U = 100$, is paid at the contract inception
- the automatic rebalancing mechanism acts at each anniversary of the contract
- a guarantee is embedded into the contract assuring that the low-volatility fund accrues interest at the target rate

The contract

- The rationale of the contract is to include in the same policy two different levels of risk. An investment with small risk that aims at preserving the single premium and, at the same time, an investment with an higher risk to capture the opportunity of greater returns furnished by more volatile markets
- for illustrative purposes, we consider a policy with maturity $T = 10$ years, target date $\tau = 5$ years, and target interest rate of 3.5% per annum
- a single premium, $U = 100$, is paid at the contract inception
- the automatic rebalancing mechanism acts at each anniversary of the contract
- a guarantee is embedded into the contract assuring that the low-volatility fund accrues interest at the target rate
- the sum invested in the low-volatility fund is $100 \times (1 + 0.035)^{-5} = 84.2$, while the sum invested in the high-volatility fund is $100 - 84.2 = 15.8$

The contract

| year | LV fund return | LV fund value | HV fund return | HV fund value | total policy value |
|------|-------------------|------------------|-------------------|------------------|-----------------------|
| 0 | - | 84.2 | - | 15.8 | 100 |
| 1 | 5.2% | 87.14 | 2.8% | 16.63 | 103.77 |
| 2 | 4.8% | 90.19 | 6.4% | 17.74 | 107.93 |
| 3 | 3.2% | 93.35 | 4% | 17.06 | 110.41 |
| 4 | 1.8% | 96.62 | 2.7% | 14.81 | 111.43 |
| 5 | -0.5% | 100.00 | 3.5% | 10.35 | 110.35 |
| 6 | -1.2% | 100.00 | 1.3% | 8.19 | 108.19 |
| 7 | -2.7% | 100.00 | -4% | 4.11 | 104.11 |
| 8 | -3% | 100.00 | -8% | 0 | 100 |
| 9 | 3.2% | 100.00 | - | 2.17 | 102.17 |
| 10 | 4.2% | 104.2 | 5% | 2.28 | 106.48 |

This table reports an example regarding to a possible scenario of the evolution of the policy value

The evaluation framework

- We denote the investment funds value at time t , $F_L(t)$ and $F_H(t)$, respectively

The evaluation framework

- We denote the investment funds value at time t , $F_L(t)$ and $F_H(t)$, respectively

$$dF_L(t) = F_L(t)(rdt + \sigma_L dZ_L(t)), \quad F_L(t_k^+) = F_L(t_k) + D_k$$

The evaluation framework

- We denote the investment funds value at time t , $F_L(t)$ and $F_H(t)$, respectively

$$dF_L(t) = F_L(t)(rdt + \sigma_L dZ_L(t)), \quad F_L(t_k^+) = F_L(t_k) + D_k$$

$$dF_H(t) = F_H(t)(rdt + \sigma_H dZ_H(t)), \quad F_H(t_k^+) = F_H(t_k) - D_k,$$

The evaluation framework

- We denote the investment funds value at time t , $F_L(t)$ and $F_H(t)$, respectively

$$dF_L(t) = F_L(t)(rdt + \sigma_L dZ_L(t)), \quad F_L(t_k^+) = F_L(t_k) + D_k$$

$$dF_H(t) = F_H(t)(rdt + \sigma_H dZ_H(t)), \quad F_H(t_k^+) = F_H(t_k) - D_k,$$

$$D_k = F_L(t_{k-1}^+) \min(1 + i_g - (1 - \alpha I_{\{t=t_k\}})(1 + r_L(t_k)), 0) \\ + \min(F_L(t_{k-1}^+) \max(1 + i_g - (1 - \alpha I_{\{t=t_k\}})(1 + r_L(t_k)), 0), F_H(t_k)),$$

The evaluation framework

- We denote the investment funds value at time t , $F_L(t)$ and $F_H(t)$, respectively

$$dF_L(t) = F_L(t)(rdt + \sigma_L dZ_L(t)), \quad F_L(t_k^+) = F_L(t_k) + D_k$$

$$dF_H(t) = F_H(t)(rdt + \sigma_H dZ_H(t)), \quad F_H(t_k^+) = F_H(t_k) - D_k,$$

$$D_k = F_L(t_{k-1}^+) \min(1 + i_g - (1 - \alpha I_{\{t=t_k\}})(1 + r_L(t_k)), 0) \\ + \min(F_L(t_{k-1}^+) \max(1 + i_g - (1 - \alpha I_{\{t=t_k\}})(1 + r_L(t_k)), 0), F_H(t_k)),$$

- r is the risk-free force of interest

The evaluation framework

- We denote the investment funds value at time t , $F_L(t)$ and $F_H(t)$, respectively

$$dF_L(t) = F_L(t)(rdt + \sigma_L dZ_L(t)), \quad F_L(t_k^+) = F_L(t_k) + D_k$$

$$dF_H(t) = F_H(t)(rdt + \sigma_H dZ_H(t)), \quad F_H(t_k^+) = F_H(t_k) - D_k,$$

$$D_k = F_L(t_{k-1}^+) \min(1 + i_g - (1 - \alpha I_{\{t=t_k\}})(1 + r_L(t_k)), 0) \\ + \min(F_L(t_{k-1}^+) \max(1 + i_g - (1 - \alpha I_{\{t=t_k\}})(1 + r_L(t_k)), 0), F_H(t_k)),$$

- r is the risk-free force of interest
- $Z_L(t)$ and $Z_H(t)$ are two standard Brownian motions with correlation ρ

The evaluation framework

- We denote the investment funds value at time t , $F_L(t)$ and $F_H(t)$, respectively

$$dF_L(t) = F_L(t)(rdt + \sigma_L dZ_L(t)), \quad F_L(t_k^+) = F_L(t_k) + D_k$$

$$dF_H(t) = F_H(t)(rdt + \sigma_H dZ_H(t)), \quad F_H(t_k^+) = F_H(t_k) - D_k,$$

$$D_k = F_L(t_{k-1}^+) \min(1 + i_g - (1 - \alpha I_{\{t=t_k\}})(1 + r_L(t_k)), 0) \\ + \min(F_L(t_{k-1}^+) \max(1 + i_g - (1 - \alpha I_{\{t=t_k\}})(1 + r_L(t_k)), 0), F_H(t_k)),$$

- r is the risk-free force of interest
- $Z_L(t)$ and $Z_H(t)$ are two standard Brownian motions with correlation ρ
- α is the annual fee, I is the indicator function and $r_L(t_k)$ is the rate of return of the low-investment fund in the time interval $[t_{k-1}^+, t_k]$

The evaluation framework

- We evaluate at fair rates the policy in the binomial framework of Cox, Ross, and Rubinstein

The evaluation framework

- We evaluate at fair rates the policy in the binomial framework of Cox, Ross, and Rubinstein
- the policy time to maturity is divided into N time intervals of length $\Delta t = T/N$

The evaluation framework

- We evaluate at fair rates the policy in the binomial framework of Cox, Ross, and Rubinstein
- the policy time to maturity is divided into N time intervals of length $\Delta t = T/N$
- at the end of each time interval, the low (high)-volatility fund value may jump up by the factor $u_L = e^{\sigma_L \sqrt{\Delta t}}$ ($u_H = e^{\sigma_H \sqrt{\Delta t}}$), or it may jump down by the factor $d_L = 1/u_L$ ($d_H = 1/u_H$)

The evaluation framework

- We evaluate at fair rates the policy in the binomial framework of Cox, Ross, and Rubinstein
- the policy time to maturity is divided into N time intervals of length $\Delta t = T/N$
- at the end of each time interval, the low (high)-volatility fund value may jump up by the factor $u_L = e^{\sigma_L \sqrt{\Delta t}}$ ($u_H = e^{\sigma_H \sqrt{\Delta t}}$), or it may jump down by the factor $d_L = 1/u_L$ ($d_H = 1/u_H$)
- the risk-neutral probability of an upward movement of the fund with low (high) volatility is $q_L = (e^{r\Delta t} - d_L)/(u_L - d_L)$ ($q_H = (e^{r\Delta t} - d_H)/(u_H - d_H)$)

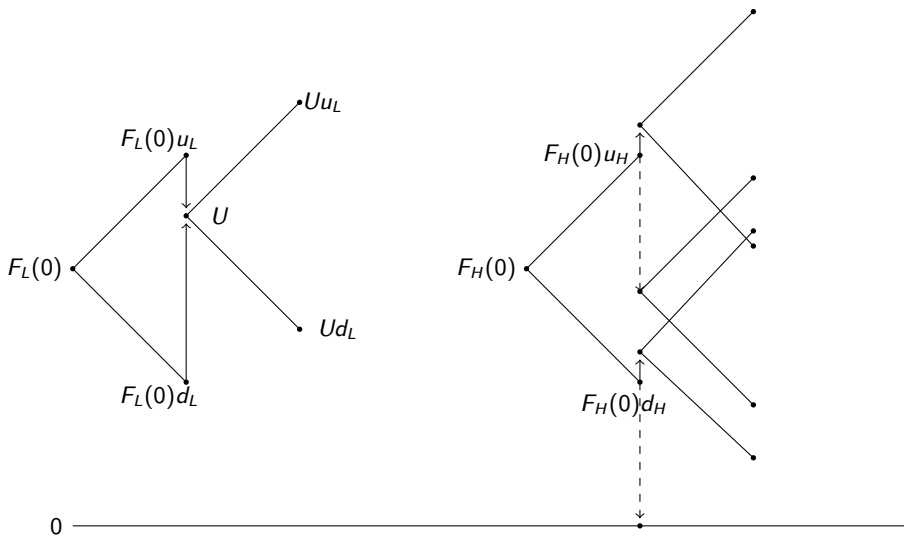
The evaluation framework

- We evaluate at fair rates the policy in the binomial framework of Cox, Ross, and Rubinstein
- the policy time to maturity is divided into N time intervals of length $\Delta t = T/N$
- at the end of each time interval, the low (high)-volatility fund value may jump up by the factor $u_L = e^{\sigma_L \sqrt{\Delta t}}$ ($u_H = e^{\sigma_H \sqrt{\Delta t}}$), or it may jump down by the factor $d_L = 1/u_L$ ($d_H = 1/u_H$)
- the risk-neutral probability of an upward movement of the fund with low (high) volatility is $q_L = (e^{r\Delta t} - d_L)/(u_L - d_L)$ ($q_H = (e^{r\Delta t} - d_H)/(u_H - d_H)$)
- the joint probabilities are computed following the approach of Hull and White

The evaluation framework

- We evaluate at fair rates the policy in the binomial framework of Cox, Ross, and Rubinstein
- the policy time to maturity is divided into N time intervals of length $\Delta t = T/N$
- at the end of each time interval, the low (high)-volatility fund value may jump up by the factor $u_L = e^{\sigma_L \sqrt{\Delta t}}$ ($u_H = e^{\sigma_H \sqrt{\Delta t}}$), or it may jump down by the factor $d_L = 1/u_L$ ($d_H = 1/u_H$)
- the risk-neutral probability of an upward movement of the fund with low (high) volatility is $q_L = (e^{r\Delta t} - d_L)/(u_L - d_L)$ ($q_H = (e^{r\Delta t} - d_H)/(u_H - d_H)$)
- the joint probabilities are computed following the approach of Hull and White

$$q_{uu} = q_L q_H + \frac{\rho}{4}, \quad q_{ud} = q_L(1 - q_H) - \frac{\rho}{4},$$
$$q_{du} = (1 - q_L)q_H - \frac{\rho}{4}, \quad q_{dd} = (1 - q_L)(1 - q_H) + \frac{\rho}{4}.$$



An example of the fund dynamics with two time steps

The evaluation framework

The low-volatility fund

- We assume that each time interval between two consecutive remix dates, $t_k - t_{k-1}$ ($k = 1, \dots, m$), is divided into h time steps

The evaluation framework

The low-volatility fund

- We assume that each time interval between two consecutive remix dates, $t_k - t_{k-1}$ ($k = 1, \dots, m$), is divided into h time steps
- we label each node of the low-volatility fund lattice by a couple (i, j)

The evaluation framework

The low-volatility fund

- We assume that each time interval between two consecutive remix dates, $t_k - t_{k-1}$ ($k = 1, \dots, m$), is divided into h time steps
- we label each node of the low-volatility fund lattice by a couple (i, j)
- starting from $G(0) = F_L(0)$, at each remix date the low-volatility fund is reset to the guaranteed level, that is

$$G(k) = \begin{cases} F_L(0)e^{ig t_k} & \text{if } 0 < t_k < \tau \\ U & \text{if } t_k \geq \tau. \end{cases}$$

The evaluation framework

The low-volatility fund

- We assume that each time interval between two consecutive remix dates, $t_k - t_{k-1}$ ($k = 1, \dots, m$), is divided into h time steps
- we label each node of the low-volatility fund lattice by a couple (i, j)
- starting from $G(0) = F_L(0)$, at each remix date the low-volatility fund is reset to the guaranteed level, that is

$$G(k) = \begin{cases} F_L(0)e^{i_g t_k} & \text{if } 0 < t_k < \tau \\ U & \text{if } t_k \geq \tau. \end{cases}$$

- the low-volatility fund may assume one of the values

$$F_L(i, j) = G(\lceil i/h \rceil - 1)u_L^{i - (\lceil i/h \rceil - 1)h - j}d_L^j(1 - \alpha I_{\{i=kh\}}),$$

$$i = 0, \dots, N, \quad j = 0, \dots, \bar{j}_L(i) \equiv i - (\lceil i/h \rceil - 1)h.$$

The evaluation framework

The high-volatility fund

- The high-volatility lattice lacks its reconnecting structure at the first remix date, and the evaluation model becomes readily unmanageable from a computational point of view

The evaluation framework

The high-volatility fund

- The high-volatility lattice lacks its reconnecting structure at the first remix date, and the evaluation model becomes readily unmanageable from a computational point of view
- to overcome this obstacle a set of “representative” values of the high-volatility fund is generated at each time step

The evaluation framework

The high-volatility fund

- The high-volatility lattice lacks its reconnecting structure at the first remix date, and the evaluation model becomes readily unmanageable from a computational point of view
- to overcome this obstacle a set of “representative” values of the high-volatility fund is generated at each time step
- at first, we consider the maximum and the minimum possible values of the high volatility fund at each time step

The evaluation framework

The high-volatility fund

- The high-volatility lattice lacks its reconnecting structure at the first remix date, and the evaluation model becomes readily unmanageable from a computational point of view
- to overcome this obstacle a set of “representative” values of the high-volatility fund is generated at each time step
- at first, we consider the maximum and the minimum possible values of the high volatility fund at each time step
- the maximum possible high-volatility fund value at time $i\Delta t$ is

$$F_H^M(i) = (F_H^M(i-1) + [F_L(i-1, 0) - G(\lceil i/h \rceil - 1)] I_{\{i-1=kh\}}) u_H(1 - \alpha I_{\{i=kh\}}),$$

The evaluation framework

The high-volatility fund

- The high-volatility lattice lacks its reconnecting structure at the first remix date, and the evaluation model becomes readily unmanageable from a computational point of view
- to overcome this obstacle a set of “representative” values of the high-volatility fund is generated at each time step
- at first, we consider the maximum and the minimum possible values of the high volatility fund at each time step

- the maximum possible high-volatility fund value at time $i\Delta t$ is

$$F_H^M(i) = (F_H^M(i-1) + [F_L(i-1, 0) - G(\lceil i/h \rceil - 1)] I_{\{i-1=kh\}}) u_H (1 - \alpha I_{\{i=kh\}}),$$

- the minimum possible high-volatility fund value at time $i\Delta t$ is

The evaluation framework

The high-volatility fund

- The high-volatility lattice lacks its reconnecting structure at the first remix date, and the evaluation model becomes readily unmanageable from a computational point of view
- to overcome this obstacle a set of “representative” values of the high-volatility fund is generated at each time step
- at first, we consider the maximum and the minimum possible values of the high volatility fund at each time step
- the maximum possible high-volatility fund value at time $i\Delta t$ is

$$F_H^M(i) = (F_H^M(i-1) + [F_L(i-1, 0) - G(\lceil i/h \rceil - 1)] I_{\{i-1=kh\}}) u_H (1 - \alpha I_{\{i=kh\}}),$$

- the minimum possible high-volatility fund value at time $i\Delta t$ is

$$F_H^m(i) = \max(F_H^m(i-1) - [G(\lceil i/h \rceil - 1) - F_L(i-1, \bar{j}_L(i-1))] I_{\{i-1=kh\}}, 0) d_H (1 - \alpha I_{\{i=kh\}})$$

The evaluation framework

The high-volatility fund

- The representative values of the high-volatility fund are labeled $F_H(i, l)$, $i = 0, \dots, N$, $l = 0, \dots, \bar{l}_H(i)$,

The evaluation framework

The high-volatility fund

- The representative values of the high-volatility fund are labeled $F_H(i, l)$, $i = 0, \dots, N$, $l = 0, \dots, \bar{l}_H(i)$,
- the first value in the set at the i -th time step is $F_H(i, 0) = F_H^M(i)$

The evaluation framework

The high-volatility fund

- The representative values of the high-volatility fund are labeled $F_H(i, l)$, $i = 0, \dots, N$, $l = 0, \dots, \bar{l}_H(i)$,
- the first value in the set at the i -th time step is $F_H(i, 0) = F_H^M(i)$
- the last value in the set at the i -th time step is $F_H(i, \bar{l}_H(i)) = F_H^m(i)$

The evaluation framework

The high-volatility fund

- The representative values of the high-volatility fund are labeled $F_H(i, l)$, $i = 0, \dots, N$, $l = 0, \dots, \bar{l}_H(i)$,
- the first value in the set at the i -th time step is $F_H(i, 0) = F_H^M(i)$
- the last value in the set at the i -th time step is $F_H(i, \bar{l}_H(i)) = F_H^m(i)$
- for the first h time steps the remaining representative values coincide with those generated by the recombining binomial tree describing the evolution of the high-volatility fund

The evaluation framework

The high-volatility fund

- The representative values of the high-volatility fund are labeled $F_H(i, l)$, $i = 0, \dots, N$, $l = 0, \dots, \bar{l}_H(i)$,
- the first value in the set at the i -th time step is $F_H(i, 0) = F_H^M(i)$
- the last value in the set at the i -th time step is $F_H(i, \bar{l}_H(i)) = F_H^m(i)$
- for the first h time steps the remaining representative values coincide with those generated by the recombining binomial tree describing the evolution of the high-volatility fund
- from time step $h + 1$ onward, the remaining representative values are fictitious values generated in such a way that the difference between two consecutive fund values is proportional to $F_H(0)\Delta t$

The evaluation framework

- The policy value at maturity is set equal to
$$W(N, j, l) = F_L(N, j) + F_H(N, l), j = 0, \dots, \bar{j}_L(N), l = 0, \dots, \bar{l}_H(N)$$

The evaluation framework

- The policy value at maturity is set equal to
$$W(N, j, l) = F_L(N, j) + F_H(N, l), j = 0, \dots, \bar{j}_L(N), l = 0, \dots, \bar{l}_H(N)$$
- the policy value at inception is computed through the usual backward induction procedure coupled with linear interpolation

The evaluation framework

- The policy value at maturity is set equal to
$$W(N, j, l) = F_L(N, j) + F_H(N, l), j = 0, \dots, \bar{j}_L(N), l = 0, \dots, \bar{l}_H(N)$$
- the policy value at inception is computed through the usual backward induction procedure coupled with linear interpolation
- once the policy value at inception, $W(\alpha)$, has been obtained, the fee value that makes the policy actuarially fair, is computed by solving the non-linear equation $W(\alpha) = U$

The evaluation framework

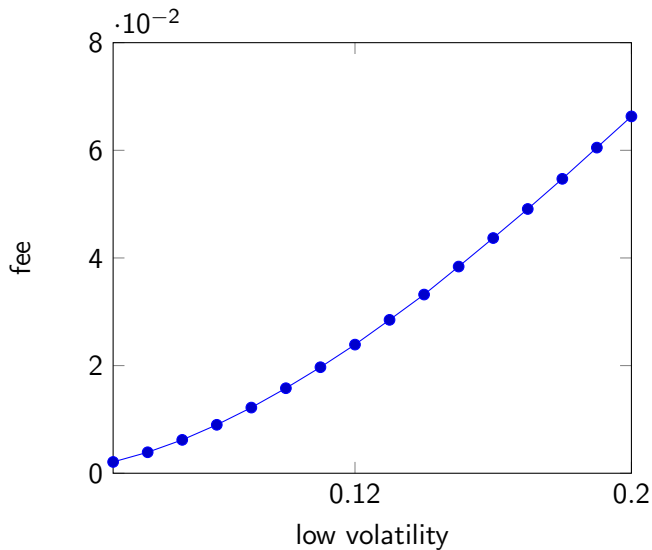
- The policy value at maturity is set equal to
$$W(N, j, l) = F_L(N, j) + F_H(N, l), j = 0, \dots, \bar{j}_L(N), l = 0, \dots, \bar{l}_H(N)$$
- the policy value at inception is computed through the usual backward induction procedure coupled with linear interpolation
- once the policy value at inception, $W(\alpha)$, has been obtained, the fee value that makes the policy actuarially fair, is computed by solving the non-linear equation $W(\alpha) = U$
- this can be done through common numerical root-finding schemes

Numerical Results

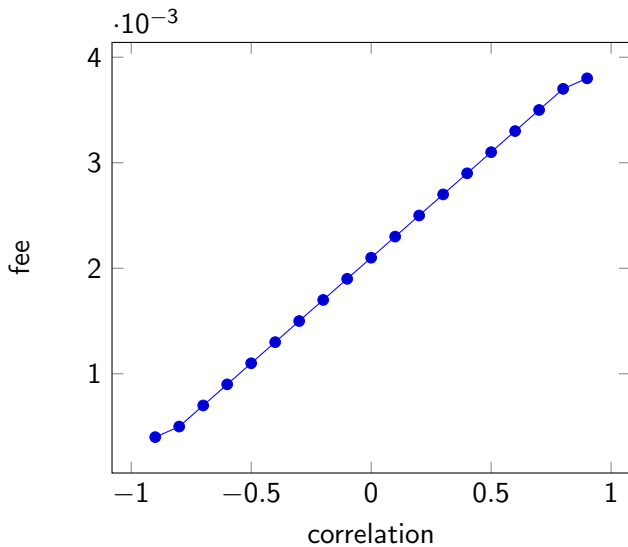
| | T | | | | |
|------|----------|----------|----------|----------|----------|
| | 2 | 4 | 6 | 8 | 10 |
| L | 100.7844 | 100.6940 | 100.5748 | 100.4954 | 100.4483 |
| MC | 100.7858 | 100.6939 | 100.5754 | 100.4836 | 100.4309 |
| s.e. | (0.0042) | (0.0077) | (0.0123) | (0.0183) | (0.0265) |
| fee | 0.0108 | 0.0021 | 0.0012 | 0.0008 | 0.0006 |

in this table we consider policies with maturities, $T = 2, 4, 6, 8, 10$ years. In all cases, the target date, τ , is set equal to $T/2$, and the target rate, i_g , is 3.5% per year. The remaining parameters are set as follows: the continuously compounded risk-free interest rate is $r = 4\%$ per year, the high volatility is $\sigma_H = 0.3$, the low volatility is $\sigma_L = 0.05$, and the correlation is $\rho = 0$. The row labeled L contains the policy values at inception computed by the proposed model. The row labeled MC reports the corresponding values obtained by Monte Carlo simulations, while in the row labeled s.e. the standard error is reported in brackets. The last row, labeled fee, contains the fair fees computed with the proposed method.

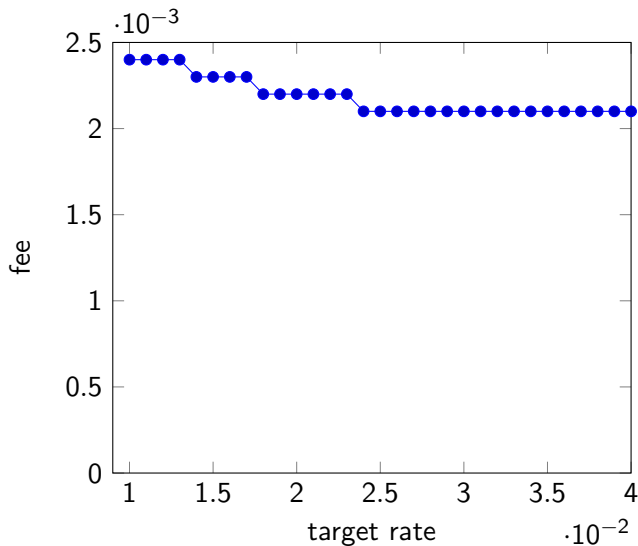
Numerical Results



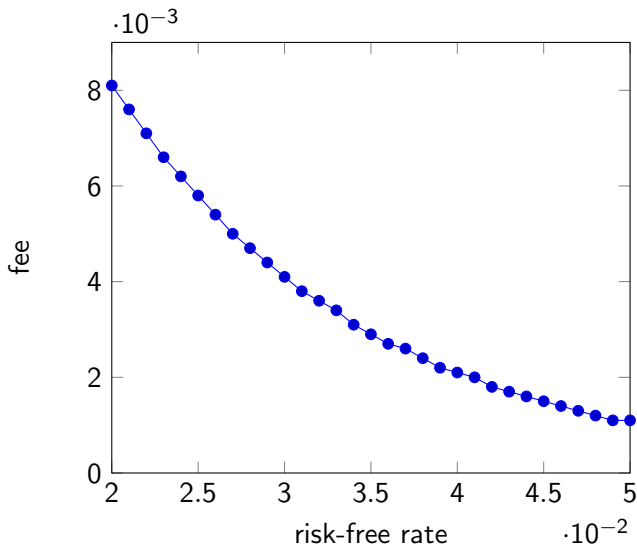
Numerical Results



Numerical Results



Numerical Results



Thank You for Your attention!