Analysing systematic risk and diversification benefits

Weihao Choo

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Lyon, France

weihao.choo@au.ey.com

chooweihao@gmail.com

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References

Contents of this presentation are mainly based on three papers:

- Choo, W., & De Jong, P. (2016). Insights to systematic risk and diversification across a joint probability distribution. *Insurance: Mathematics and Economics*, *67*, 142-150.
- Choo, W., & De Jong, P. (2010). Determining and allocating diversification benefits for a portfolio of risks. *Astin Bulletin*, *40*(01), 257-269.
- Choo, W. and De Jong, P. (2009). Loss reserving using loss aversion functions. *Insurance: Mathematics and Economics,* 45(2), pp.271-277

Motivation

- Diversification benefits are typical when modelling the aggregate risk of imperfectly dependent random component losses.
- The total diversification benefit is allocated back to component losses, e.g. for performance measurement.
- Key questions usually are:
  - How I explain the allocated diversification benefits?
  - Which "part" of the joint distribution is contributing to the benefits?
  - How does the dependence structure affect the benefits?
- A formal approach is proposed to answer the above questions.
- An illustration is used to show the effectiveness of the proposed approach.

Standalone risk

- Consider a random component loss $x > 0$ with distribution function F and percentile rank $u=F(x)$ which is uniformly distributed on (0,1).
- The standalone risk of a loss $x>0$ is measured by distortion:

$$r = \int_0^\infty 1 - \Phi\{F(x)\}\mathrm{d}x - \mathrm{E}(x) = \mathrm{cov}\{x, \phi(u)\}$$

- In the above Φ is a distortion function and $\phi = \Phi'$ is an aversion function.
- Φ is convex and ϕ is non-decreasing.
- Common examples of distortion risks are the conditional-tail-expectation or expected shortfall, expected-maximal-loss and proportional hazards risk.
- Distortion risks are coherent and most importantly, sub-additive.

Systematic risk

- Suppose x is aggregated with several component losses, e.g. losses from multiple insurance classes or business units.
- The total loss is x_+. The distribution function of x_+ is F_+ and $u_+=F_+(x_+)$.
- Then the aggregate risk, again using the distortion approach, is

$$r_+ = \mathrm{cov}\left\{x_+, \phi(u_+)\right\}$$

- The systematic risk of x, using the well known Euler principle, is given by

$$\bar{r} = \mathrm{cov}\left\{x, \phi(u_+)\right\}$$

- The systematic risk of x is also the portion of r_+ allocated to x.
- Adding up systematic risks across all component losses gives r_+.

Diversification benefits

- The Euler principle gives several desirable practical properties, in particular non-negative allocated diversification benefits for all component losses.
- Therefore the standalone risk *r* of *x* can be split into systematic risk and a diversification benefit which are both non-negative:

$$\bar{r}, \quad r - \bar{r} \geq 0$$

- Key focus is on the systematic risk ratio:

$$\theta = \frac{\bar{r}}{r} = \frac{\operatorname{cov}\{x, \phi(u_+)\}}{\operatorname{cov}\{x, \phi(u)\}} \leq 1$$

- A systematic risk ratio $\theta = 1$ indicates zero diversification benefit for *x*.
- As θ decreases, the diversification benefit for *x* increases and vice versa.
- θ decreases when *x* becomes more weakly dependent on x_+ (intuitive).

VaR layers

- It is well known that any $x > 0$ can be decomposed into infinitesimal layers:

$$x = \int_0^1 I(x > k)\mathrm{d}k$$

- Here $I(x>k)\mathrm{d}k$ is the k-layer of x, or the portion $[k,k+\mathrm{d}k]$ of x.
- Layers are common quantity in excess-of-loss reinsurance and asset-backed securities (called tranches instead of layers).
- By substituting $k=F^{-1}(\alpha)=V(\alpha)$ where $V(\alpha)$ is the Value-at-Risk or VaR of x at threshold α:

$$x = \int_0^1 I(u > \alpha)V'(\alpha)\mathrm{d}\alpha$$

- Now $I(u>\alpha)V'(\alpha)\,\mathrm{d}\alpha$ is the **α–VaR layer** of x.
- Now loss x is decomposed into infinitesimal **VaR layers.**

Standalone and systematic risk of VaR layers

- Using the VaR-layer decomposition of *x*, we can rewrite the standalone and systematic risks of *x* as:

$$r = \int_0^1 \operatorname{cov}\{I(u > \alpha)V'(a), \phi(u)\} \mathrm{d}\alpha, \qquad \bar{r} = \int_0^1 \operatorname{cov}\{I(u > \alpha)V'(a), \phi(u_+)\} \mathrm{d}\alpha$$

- Hence the standalone risk **density** of *x* at the α–VaR layer is:

$$r_\alpha = V'(a) \operatorname{cov}\{I(u > \alpha), \phi(u)\}$$

- And the systematic risk **density** of *x* at the α–VaR layer is:

$$\bar{r}_\alpha = V'(a) \operatorname{cov}\{I(u > \alpha), \phi(u_+)\}$$

- The diversification benefit of the α–VaR layer of *x* is the difference between the two risk densities.

Systematic risk ratio of VaR layers

- The systematic risk ratio of the α–VaR layer of x is:

$$\theta_\alpha = \frac{V'(a)\,\mathrm{cov}\{I(u>\alpha),\phi(u_+)\}}{V'(a)\,\mathrm{cov}\{I(u>\alpha),\phi(u)\}} = \frac{\mathrm{cov}\{I(u>\alpha),\phi(u_+)\}}{\mathrm{cov}\{I(u>\alpha),\phi(u)\}} \le 1$$

- θ_α measures the portion of the risk of the α–VaR layer which is remaining after diversification with other component losses.
- There is zero diversification between VaR layers of x as these layers are all comonotonic, and distortion risks are additive under comonotonicity.
- Therefore the diversification at the α–VaR layer of x is negatively related to its dependence with x_+ (after application of F_+ and ϕ).
- This dependence is also the **local dependence** between x at α–VaR and u_+.

Systematic risk ratio of VaR layers

- The overall systematic risk ratio of *x* can be rewritten as:

$$\theta = \frac{\bar{r}}{r} = \frac{\int_0^1 r_\alpha \theta_\alpha \mathrm{d}\alpha}{\int_0^1 r_\alpha \mathrm{d}\alpha}$$

- Or a weighted average of systematic risk ratios of VaR layers of *x*.
- Hence the overall extent of diversification is a weighted average of the extent of diversification in individual VaR layers, which in turn is linked to local dependence between losses.
- The extent of diversification in VaR layers is driven by **local dependence**.

Recap

Quick recap before the illustration:

- We have adopted popular measures of standalone and systematic risks
- We want to analyse diversification benefits allocated to component losses
- A loss can be decomposed into infinitesimal VaR layers
- We can compute standalone, systematic and diversified risks of a VaR layer
- The decomposition allows us to granularly analyse risk and diversification
- The overall diversification is tied to a weighted average of local dependence

Illustration – setup

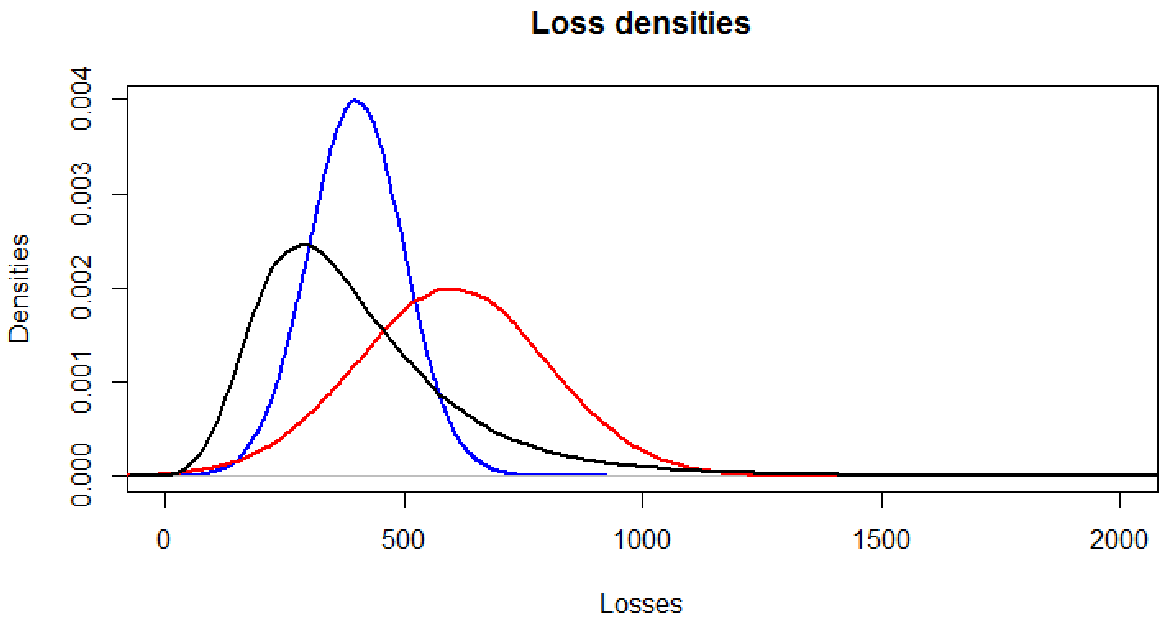
- Suppose x is the insurance loss on the personal property class and y is the insurance loss on the commercial property class.
- Both x and y comprise of an independent working loss and a comonotonic natural catastrophe loss (e.g. earthquake).
- Write:

$$x = w_x + c_x, \quad y = w_y + c_y$$

- w_x and w_y are independent working losses
- c_x and c_y are comonotonic natural catastrophe losses
- Assume working and catastrophe losses are independent.

Loss distributions

- All amounts are in millions of Euros
- $x \sim N(400,100^2)$ and $y \sim N(600,200^2)$
- $c_x=0.4c$ and $c_y=0.6c$ where c is the total catastrophe loss
- c has a Fréchet heavy tailed distribution with $\mu=300$, $\sigma=100$, $\zeta=0.1$



Risk measurement

- Let's use the following distortion function at t=0.75:

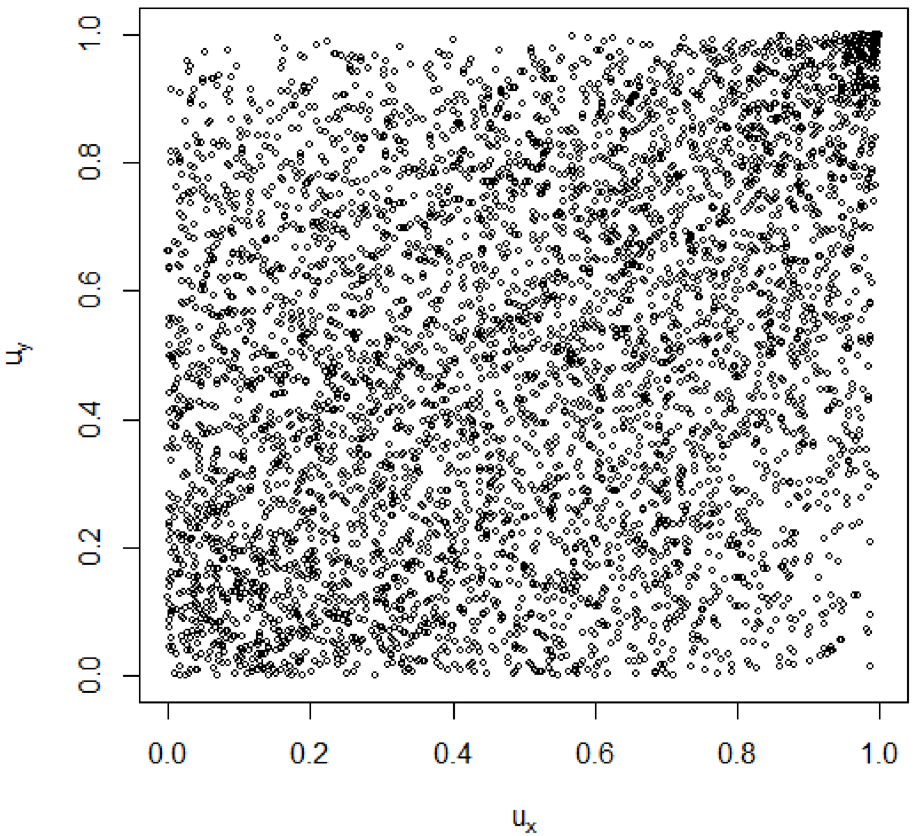
$$\Phi(u) = I(u > t)\frac{u-t}{1-t}, \quad \phi(u) = \frac{I(u > t)}{1-t}$$

- This gives the conditional-tail-expectation (or expected shortfall) measure of risk. The standalone and systematic risks of x are:

$$r = \mathrm{E}\left\{x \mid x > F^{-1}(t)\right\} - \mathrm{E}(x), \quad \bar{r} = \mathrm{E}\left\{x \mid x_+ > F_+^{-1}(t)\right\} - \mathrm{E}(x)$$

- Standalone risk of x: expected value of x given it exceeds its 75%-VaR.
- Systematic risk of x: conditional mean of x given x_+ exceeds its 75%-VaR.
- In both cases we need to subtract the mean of x to calculate risk.

Copula plot of *x* and *y*



- The copula scatterplot is mostly independent
- There is upper tail dependence due to the cat losses
- Pearson's correlation between *x* and *y* is 0.37
- Spearman's rank correlation is 0.31
- This suggests weak overall dependence between *x* and *y*
- However diversification benefits are small – see next

Standalone and systematic risks

- For loss *x* there is a 28% risk diversification:

$$r = 174, \quad \bar{r} = 125, \quad \theta = 0.72$$

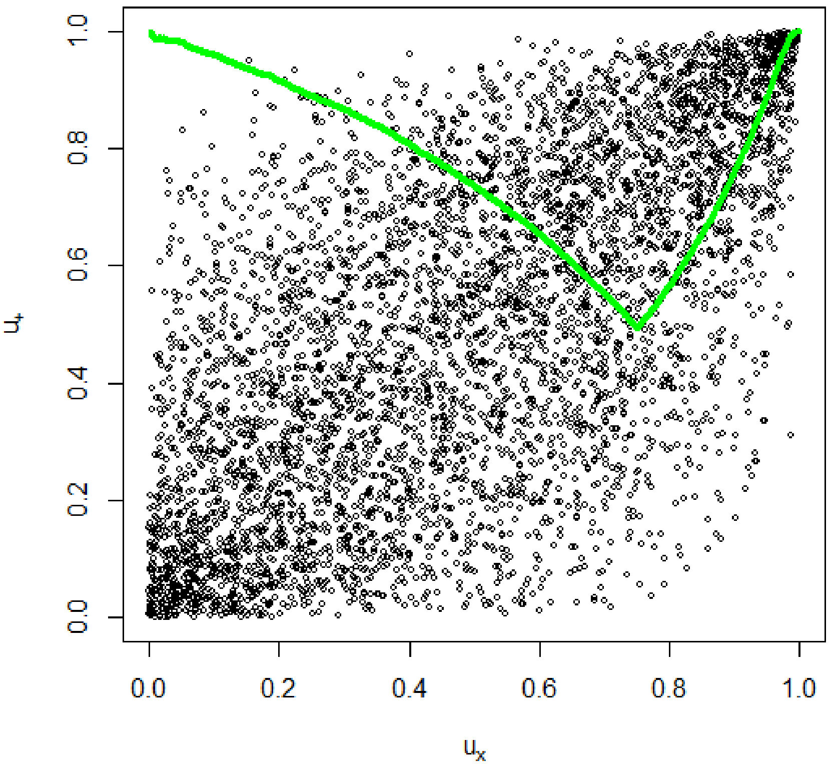
- For loss *y* there is a 8% risk diversification:

$$r = 310, \quad \bar{r} = 286, \quad \theta = 0.92$$

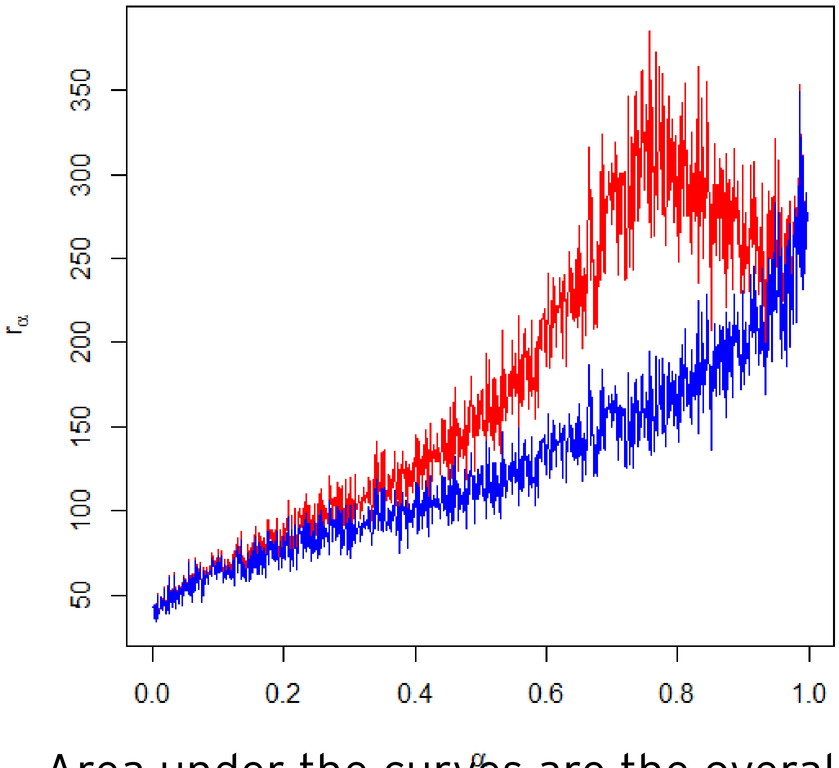
- The overall diversification is 15% although *x* and *y* are weakly correlated.
- Therefore we need to analyse diversification benefits across VaR layers to understand why this is the case.

Risks across VaR layers of *x*

Green line is the plot of θ_α against α.



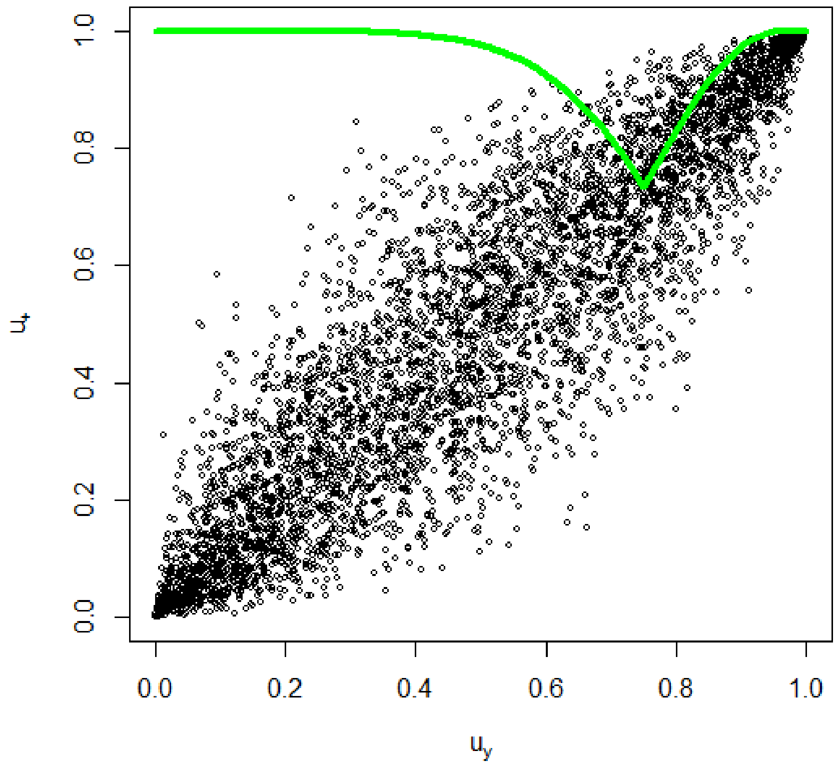
Red and blue lines are standalone and systematic risks of α–VaR layers of *x*



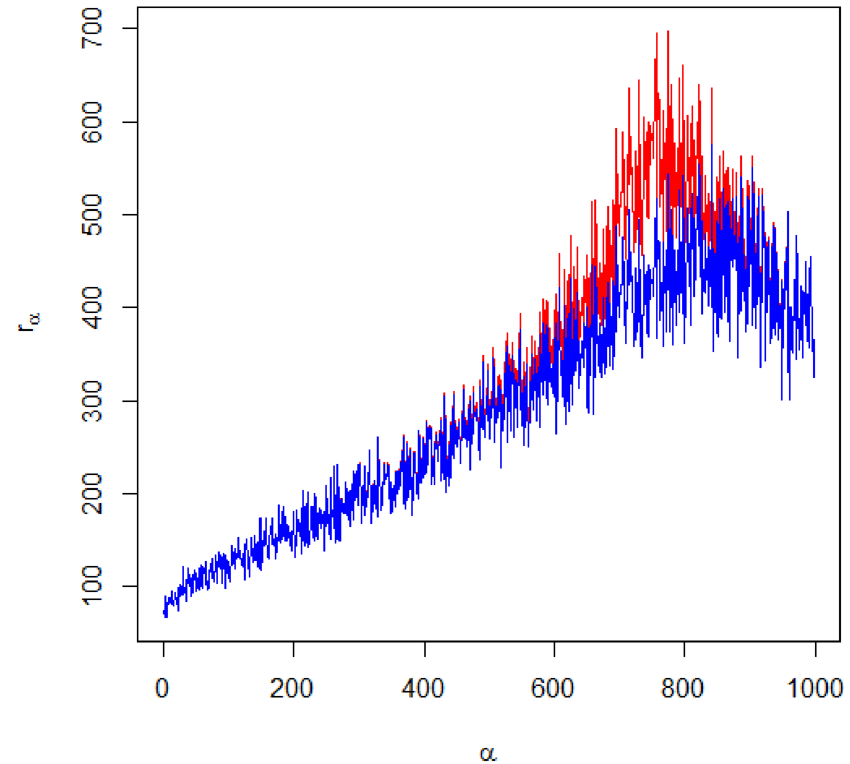
Area under the curves are the overall risks and diversification benefits.

Risks across VaR layers of *y*

Similar diagrams as previous slide, but for *y*.



Stronger dependence with the total loss especially in the tail.



Stronger local dependence leads to smaller diversification benefit for *y*.

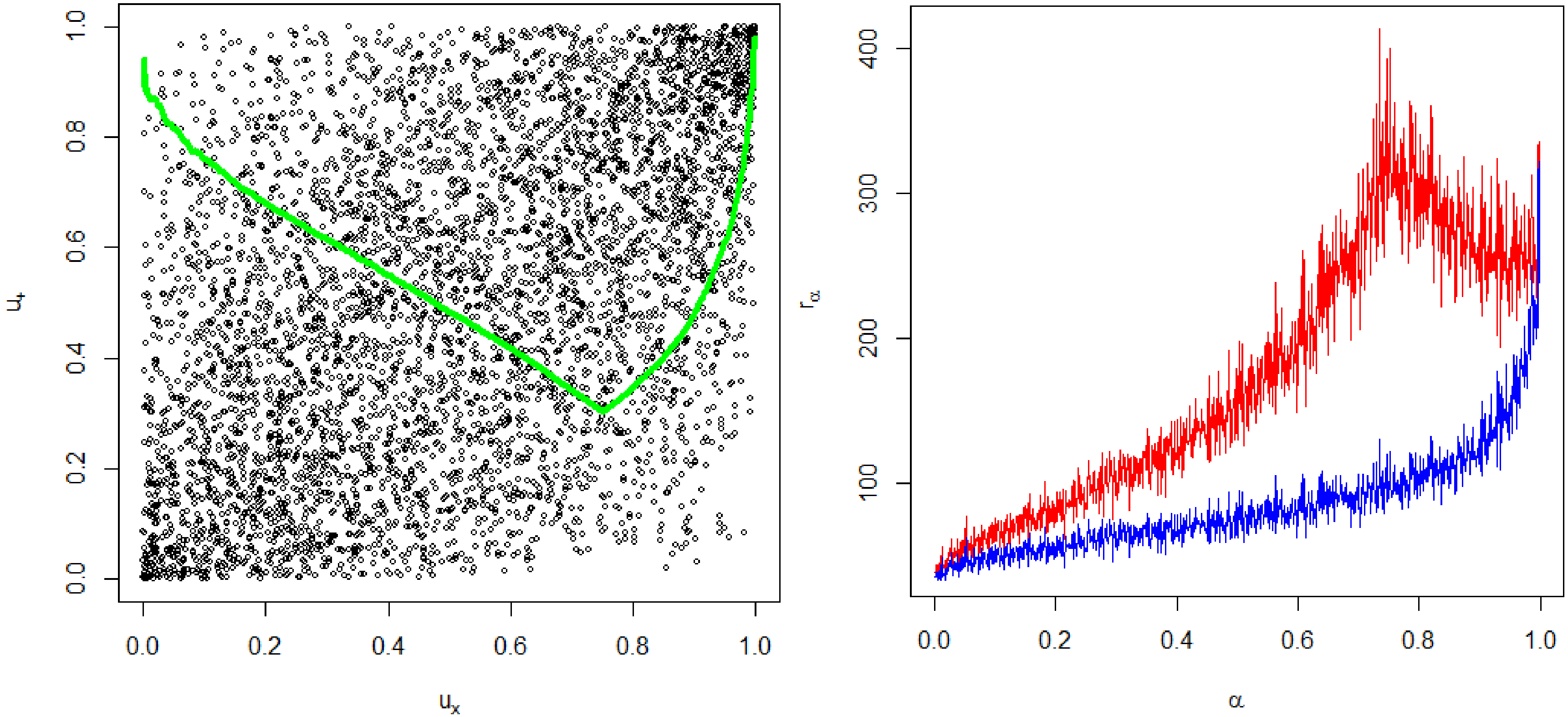
Key observations

- Using the CTE @ 75% as the risk measure, standalone risk is concentrated in the higher VaR layers of x and y as both losses are right skewed.
- Standalone risks peak at the 75% VaR layers of x and y, due to the selected risk measure.
- For x, there is a reasonable degree of diversification except for the highest VaR layers due to tail dependence.
- For y, diversification is significantly lower due to stronger dependence with the total loss. Diversification again disappears for the highest VaR layers.
- y has stronger dependence with the total loss as a result of more volatile working and catastrophe losses.

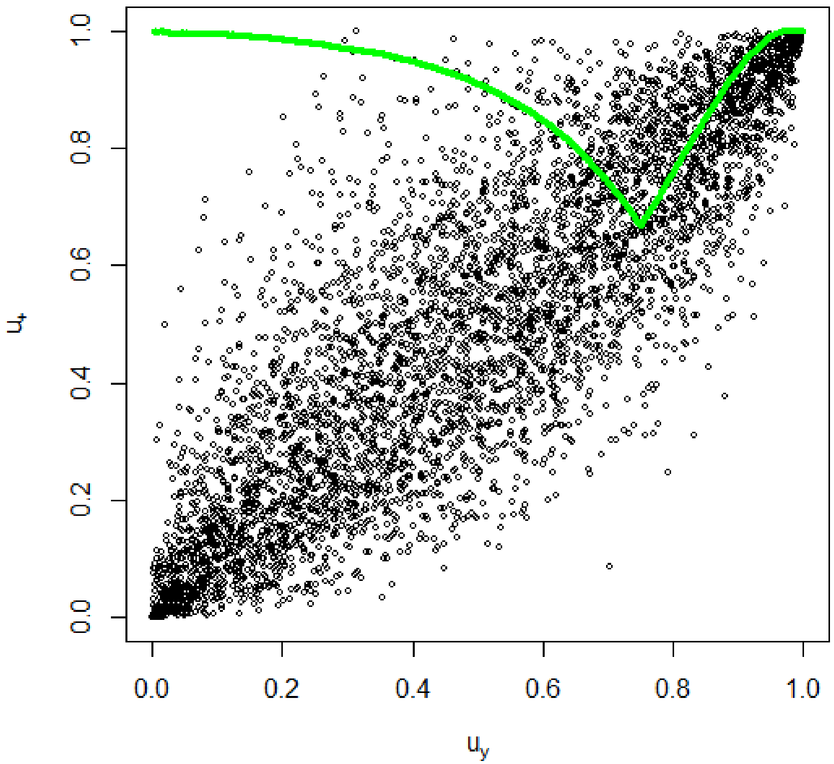
Alternative catastrophe assumptions

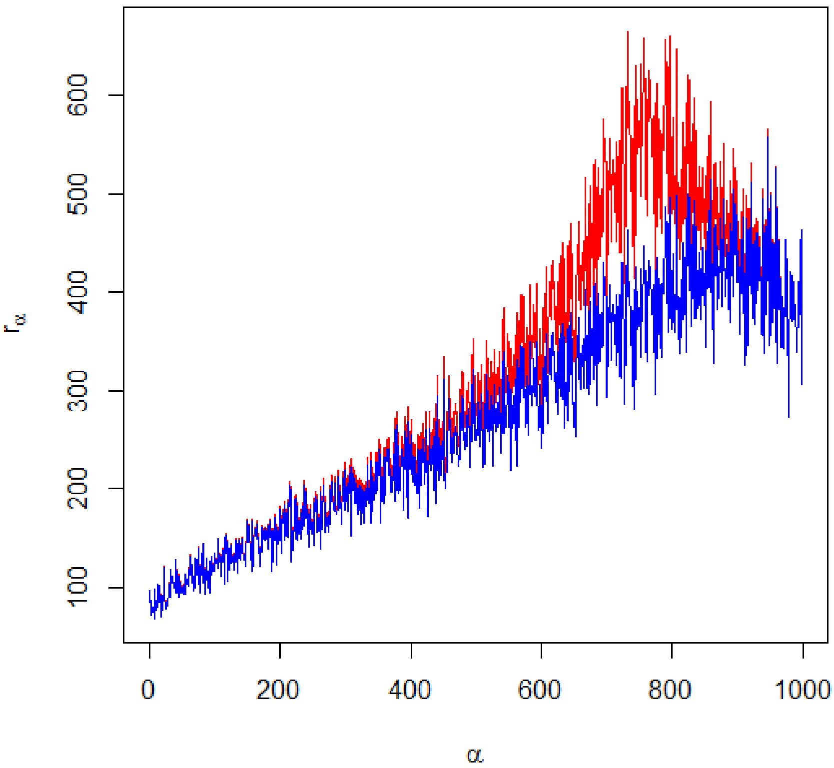
- Suppose instead c_x and c_y are independent
- Marginal distributions of *x* and *y* are unchanged
- However now *x* and *y* are independent
- Standalone risks are unchanged
- The diversification benefit for *x* increases from 28% to 51%
- The diversification benefit for *y* increases from 8% to 13%
- Overall diversification is 27% compared to 15% previously

Risks across VaR layers of *x*



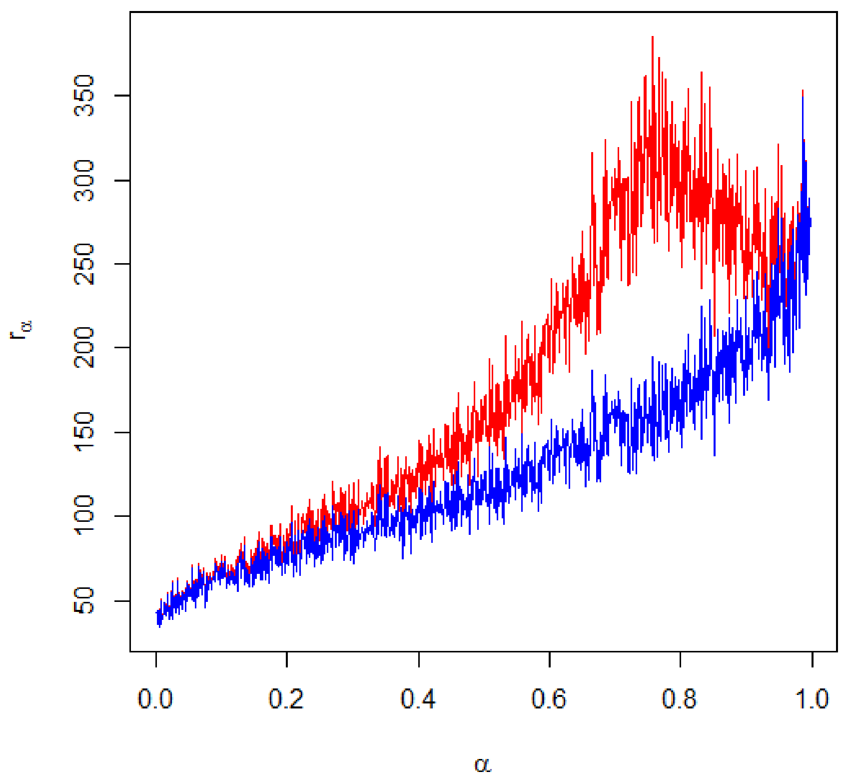
Risks across VaR layers of *y*

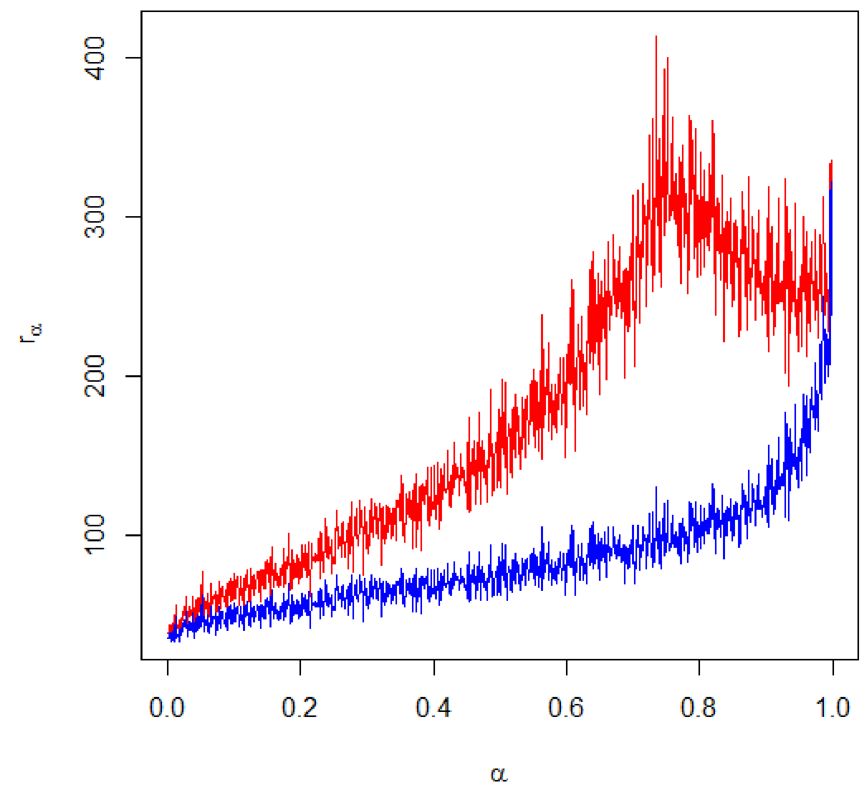




Comonotonic and independent cat losses

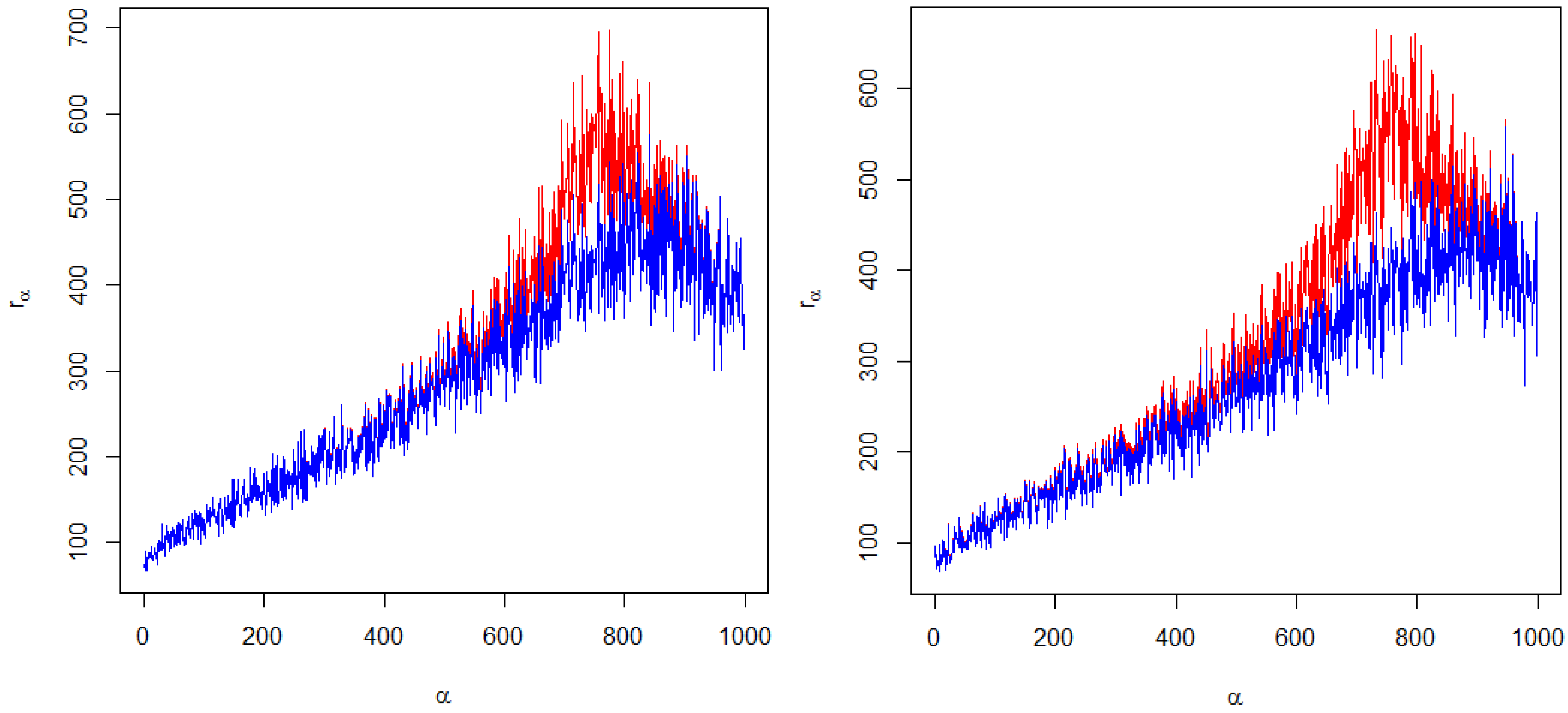
Standalone and systematic risks of VaR layers of *x* with comonotonic (left) and independent (right) cat losses.





Comonotonic and independent cat losses

Standalone and systematic risks of VaR layers of *y* with comonotonic (left) and independent (right) cat losses.



Financial markets example

- Financial markets are known to have strong tail dependence
- Tail volatility is also typically high
- This leads to low diversification despite weak overall correlation since:
  - Risks are concentrated in higher VaR layers due to tail volatility
  - Systematic risk ratios are one in high VaR layers due to tail dependence
  - As a result overall diversification is low

End