Lapse risk in life insurance: correlation and contagion effects among policyholders’ behaviors

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A word on the lapse risk

What is the lapse risk?
→ Insured exiting the portfolio due to:
   death, maturity, change of premium level, surrender, ...

Why so much interest?

1. Among the 3 major risks in life insurance;
2. Understand the behaviours, and design new products;
Lapse classification in life insurance

The 2 main historical explanations for surrenders are ([Out90])

- \textit{liquidity needs} $\rightarrow$ idiosynchratic $\rightarrow$ \textit{structural surrenders};
- \textit{economic distress} $\rightarrow$ environment $\rightarrow$ \textit{temporary surrenders}.

\textbf{Current context} : never experienced such low interest rates $\Rightarrow$ impact on the underwriting of new business...

\textbf{Threat} : massive (temporary) surrenders due to $\uparrow$ of interest rate.
Estimation of structural lapses

- **GLM** models for segmentation

- **QIS 5 (EIOPA)**: The lapse rate (LR) is based on the max b/w (1) and (2):

  **Step (1): shocks applied to structural LR (misestimation).**
  \[ LR_{up} = \min(100\%, 150\% \times LR) \rightarrow \text{our context!} \]
  \[ LR_{down} = \min(0, \max(50\% \times LR, LR - 20\%)) \].

  **Step (2): mass lapse event, \sim \text{“bank run”}.**
  30%-loss of the sum of positive surrender strain over portfolio;

- Internal model and practical approach (**S-shaped**)
Still some issues to deal with

+ Pros :
  - easy-to-understand, easy-to-implement,
  - integrates (artificially) copycat behaviours $\rightarrow$ correlation risk;

- Cons :
  - not fully realistic;
  - this is a static model...(does not depend on time $t$),
  - does not consider the contagion between policyholders...

$\Rightarrow$ We’d like to introduce a model that copes with both correlation and contagion risks to define extreme scenarios.
Dynamic contagion process for the lapse intensity

→ Extended Hawkes, [DZ11].
→ $(N_t)_{t \geq 0}$: counting process of lapses over the whole portfolio.

$$\lambda_t = \lambda_{\infty} + (\lambda_0 - \lambda_{\infty}) e^{-\beta t} + \sum_{i \geq 1} X_i e^{-\beta (t-T_i)} 1_{T_i \leq t} + \sum_{j \geq 1} Y_j e^{-\beta (t-\hat{T}_j)} 1_{\hat{T}_j \leq t}$$

(1)

where lapses occur at $T_i$, and $X_i, Y_j$ are magnitudes of jumps ($\sim$ Exp. with param. $\gamma$ and $\delta$).

1. **Structural** surrender forces $\lambda_0$, and $\lambda_{\infty}$ (constant here),
2. **Temporary** surrenders, with
   - endogenous shocks: **contagion**, internal;
   - exogenous shocks: history of $\hat{N}_t \rightarrow$ dynamic dependence, source of **correlation** in our setting (to be defined later).
Cause of correlation: interest rate movements

→ Consider a contract with

- credited rate $R^c_t$: minimum profitability + potential profit benefit.

→ Let $(r_t)_{t \geq 0}$ be the interest rate with GBM dynamics $(\mu, \sigma)$.

Q: in critical scenarios, how the surrender decision could be affected by the level of $r_t$?
Look at the following standardized spread:

\[ RG_t^0 := \frac{r_t - R_0^c}{R_0^c} \]

→ Makes sense to ↗ the propensity to lapse when \( RG_t^0 \) ↗;

→ Say that policyholders would exercise their option to surrender at time \( \hat{T}_1 \) being the first time \( RG_t^0 \) hits a constant barrier \( B > 0 \).

→ Assume that the company can then adjust the credited rate \( R_t^c \) depending on the interest rate level (to avoid massive lapses).
This defines the new standardized spread $RG_t^1$, given by

$$RG_t^1 = \frac{r_t - R_c^\hat{T}_1}{R_c^\hat{T}_1} = \frac{r_t - r_{\hat{T}_1}}{r_{\hat{T}_1}}, \quad \hat{T}_1 \leq t < \infty.$$ 

Next adjustment will be operated as soon as $RG_t^1 = B$, and so on...

$\Rightarrow$ These events thus characterize the sequence $(\hat{T}_j)_{j=0,1,...}$ s.t.

$$\hat{T}_{j+1} = \inf\{t > 0, \ RG_t^j = B\}, \quad (2)$$

with $\hat{T}_0 = 0$ for convenience.

$(\hat{N}_t = \sum_{j \geq 1} 1_{\hat{T}_j \leq t} :$ counting process associated to such events.$)$
Adjustments of the credited rate

![Chart showing interest rate dynamics and adjustments]

The chart illustrates the adjustment of the credited rate over time, with specific jump times indicated at various points. The interest rate and credited rate are depicted against time, with vertical lines marking the jump times at $T_1, T_2, T_3, T_4, T_5, T_6,$ and $T_7$.
Interest rate dynamics and adjustments

Dynamic contagion process: intensity process $\lambda_t$

Dynamic contagion process: counting process $N_t$
About external jumps at $\hat{T}_j$

→ $(r_t)_{t \geq 0}$ follows a GBM $(\log(r_t/r_0) = (\mu - \sigma^2/2)t + \sigma W_t)$.

→ $(\hat{T}_j)_{j=0,1,...}$ are hitting times of the process $RG_t$.

⇒ The events $\hat{T}_j$ can also be characterized as follows

$$\hat{T}_j = \hat{T}_{j-1} + \inf\{t \geq 0, \mu t + \sigma W_t = \log(1 + B)\}.$$

⇒ Inter-arrival times $\Delta \hat{T}_j = \hat{T}_j - \hat{T}_{j-1}$ are i.i.d., with distribution $F$ (inverse Gaussian, [SK91]):

$$\Delta \hat{T}_j \sim IG(\theta_1, \theta_2),$$

with $\theta_1 = 2 \log(1 + B)/(2\mu - \sigma^2)$, $\theta_2 = \log((1 + B)^2)/\sigma^2$. 
\( \rightarrow \lambda_t, (\lambda_t, N_t), (\lambda_t, N_t, \hat{N}_t) \) not Markovian.

\( \rightarrow \hat{T}_j = \sum_{k=1}^{j} \Delta \hat{T}_k. \)

Introduce \( P(\hat{T}_j \leq t) = F^j(t). \)

\( \rightarrow \) Denote by \( h(t) = E[\hat{N}_t] = \sum_{j=0}^{\infty} P(\hat{N}_t \geq j), \) thus

\[
h(t) = \sum_{j=0}^{\infty} F^j(t). \quad (3)
\]

The CDF \( F^j \) is still IG, with mean \( j \theta_1 \) and shape \( j \theta_2 \).
Moments of the lapse intensity

→ Assume $\beta \gamma > 1$ (stationary condition).
→ Let

\[ m(t, \theta) = E[e^{\theta \lambda t}], \]

and $m^{(n)}(t, \theta)$: $n^{th}$ derivative of $m$ with respect to $\theta$.
We have

\[ m^{(n)}(t, 0) = E[\lambda^n_t]. \]

→ Denote respectively $\xi(t, \theta)$ and $\widehat{\xi}(t, \theta)$ the m.g.f. of

\[ Z_t = \sum_{i=1}^{N_t} X_i e^{\beta T_i} \quad \text{and} \quad \widehat{Z}_t = \sum_{j=1}^{\widehat{N}_t} Y_j e^{\beta \widehat{T}_j}. \quad (4) \]

$Z_t, \widehat{Z}_t$ are discounted compound renewal processes ([LGFW10]).
Similarly, $\xi^{(n)}(t, \theta)$ and $\hat{\xi}^{(n)}(t, \theta)$ refer to the $n^{th}$ derivative $\xi(t, \theta)$ and $\hat{\xi}(t, \theta)$ with respect to $\theta$.

$\lambda_t$ can be written in the following form

$$\lambda_t = (\lambda_\infty + (\lambda_0 - \lambda_\infty)e^{-\beta t}) + e^{-\beta t}Z_t + e^{-\beta t}\hat{Z}_t.$$

⇒ We can then derive

1. the m.g.f. of $Z_t$ and $\hat{Z}_t$;
2. the m.g.f. of $\lambda_t$ in function of those of $Z_t$ and $\hat{Z}_t$;

⇒ At the end, we obtain a recursive formula.
(1) Moment generating functions of $Z_t$ and $\hat{Z}_t$

The m.g.f. $\xi$ and $\hat{\xi}$ of $Z_t$ and $\hat{Z}_t$ are given by recursive formulas:

$$E[e^{\theta Z_t}] = \xi(t, \theta) = 1 + \int_0^t \left( \frac{\theta e^{\beta u}}{\gamma - \theta e^{\beta u}} \right) \xi(t - u, \theta e^{\beta u}) m^{(1)}(u, 0) du,$$

$$E[e^{\theta \hat{Z}_t}] = \hat{\xi}(t, \theta) = 1 + \int_0^t \left( \frac{\theta e^{\beta u}}{\delta - \theta e^{\beta u}} \right) \hat{\xi}(t - u, \theta e^{\beta u}) dh(u),$$

→ We can derive the moments of the renewal processes;
→ The first moment of the intensity $\lambda_t$ is key (self-excited);
→ Recall that $h(t) = E[\hat{N}_t] = \sum_{j=0}^{\infty} F^{j*}(t).$
(2) Moment generating function of $\lambda_t$

**Proposition.** For $n > 1$, the $n^{th}$ derivative of the surrender intensity m.g.f. is given recursively:

$$m^{(n)}(t, \theta) = K(t, \lambda_0, \lambda_\infty) m^{(n-1)}(t, \theta) + \sum_{i=0}^{n-1} G(i, n) \left( l_i(t, \theta) + \tilde{l}_i(t, \theta) \right) m^{(i)}(t, \theta),$$

with $l_k$ and $J_k$ for $\{k = 1, 2, \ldots\}$ given by

$$l_k(t, \theta) = l_{k-1}^{(1)}(t, \theta) + H(l_{k-1}(t, \theta)) \xi^{(1)}(t, \theta e^{-\beta t}),$$

$$\tilde{l}_k(t, \theta) = \tilde{l}_{k-1}^{(1)}(t, \theta) + H'(l_{k-1}(t, \theta)) \tilde{\xi}^{(1)}(t, \theta e^{-\beta t}).$$
Application : expected intensity process

→ The expectation $E[\lambda_t]$ is given by

$$m^{(1)}(t, 0) = \left(\lambda_0 - \frac{\beta \lambda_{\infty}}{\beta - 1/\gamma}\right) e^{-(\beta - 1/\gamma)t} + \frac{\beta \lambda_{\infty}}{\beta - 1/\gamma} + \frac{1}{\delta} \int_0^t e^{-(\beta - 1/\gamma)(t-s)} h'(s) ds.$$  

Remark : $m^{(1)}(t, 0)$ comprises an infinite series associated with the external jumps component $(h'(s) = \sum_{j=0}^{\infty} f^{j*}(s))$.

Trick to get closed-form expressions : convolution of exponential and inverse gaussian r.v.

→ $E[N_t] = E \left[ \int_0^t \lambda_s ds \right] = \int_0^t m^{(1)}(s, 0) ds.$
Limiting behaviour of the lapse intensity

We can also compute the limit of this expectation:

\[
\lim_{t \to \infty} E[\lambda_t] = \frac{\beta \lambda_\infty}{\beta - 1/\gamma} + \frac{1}{\delta \theta_1 (\beta - 1/\gamma)}. \tag{6}
\]

→ The limiting behavior of the lapse intensity first moment strongly depends on the limit of the last term in the previous result.

→ Serfozo [2009] for such results.
Application to risk management - calibration

Some parameters can be calibrated from practitioners’ knowledge:

- $\lambda_0$ (initial force of lapse) is a constant.
  $\Rightarrow$ Exponential underlying lifetime distribution before lapse.
- $\lambda_\infty$ can be fixed by the risk managers as their goal...
  $\Rightarrow$ When the time horizon is given, this can be easily fixed.
- $B$ depicts the sensitivity of PH to opportunities (experts).

Some parameters (e.g. GBM) should be calibrated from empirical data / history whenever possible.

Others relate to the management: $\beta$ (ability to reassure the PH), $\gamma, \delta$ tie in with the mean size (SI) of lapsed contracts...
Stress tests: comparison with Solvency II and S-shaped

→ Within the Solvency II framework: run-off, 1-year horizon.
→ With regard to financial context: focus on the upper-shock.
→ Risk measures under consideration: VaR and TVaR.

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**Table** – Impact of contagion and correlation on $VaR_{\alpha}(N_t)$, $TVaR_{\alpha}(N_t)$ at level $\alpha = 99.5\%$, in a 1-year time horizon ($t = 250$).
Conclusion on stress tests

→ The shock in SII looks neither consistent nor realistic.
→ Stress tests in most of companies seem to be underestimated.

→ OK for extreme scenarios (reserving), not so realistic in classical regime (pricing).

→ Sensitivity to IR movements is obviously not linear...

→ External component has a limited impact, provided that mean size of the external jumps is low ⇒ portfolio composition is crucial!

**Perspectives :**

1. Calibration on a real-life portfolio ;
2. Retrieve the whole distribution of lapses $N_t$,
3. Extend this approach with an adapted interest rate model.
References


