Heterogeneity in a life annuity portfolio: modeling issues and risk profile assessment

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Agenda

1. Introduction & motivation
2. The age-patterns of mortality: heterogeneity issues
3. Modeling unobservable causes of heterogeneity
4. Life annuity portfolios: from homogeneity to heterogeneity
5. Heterogeneity and portfolio size: a possible trade-off?
6. Concluding remarks

Presentation based on:

- joint work with Annamaria Olivieri, University of Parma
- research work organized by the Mortality Working Group, International Actuarial Association
1 INTRODUCTION & MOTIVATION

Heterogeneity in an insured population: a classical topic in non-life actuarial mathematics

Models for observable risk factors (in particular rating factors) and non-observable risk factors

Non-observable risk factors: classical Poisson - Gamma model for the random number of claims

In life actuarial mathematics, focus on heterogeneity due to non-observable risk factors: first contribution by Beard [1959] (unfortunately an “obscure” paper, for many years unknown to actuaries!)

More contributions in demography; in particular, starting from Vaupel et al. [1979]

Recently, interest in application to life insurance field, in particular life annuities; for an extensive list of references, see Pitacco [2016]
Aims of this talk:

- to recall basic ideas about heterogeneity in age-pattern of mortality, due to observable and non-observable causes, and to focus on relevant modeling issues (Sects. 2 and 3)
  
  For more info, see [Pitacco, 2016]

- to address some features of life annuity portfolios, looking for possible risk classification criteria and consequent differentiated pricing (Sects. 4 and 5)
  
  See the paper by [Olivieri and Pitacco, 2016]
2 THE AGE-PATTERN OF MORTALITY: HETEROGENEITY ISSUES

CAUSES OF HETEROGENEITY IN A POPULATION

For each individual in the population:

- age
- gender
- health conditions
- occupation
- genetic factors
- environmental factors
- . . . . .

⇒ risk factors in the actuarial language (non-necessarily rating factors, because of legislation, market features, etc.)
The age-pattern of mortality: heterogeneity issues  \textit{(cont’d)}

Refer to a cohort, given gender

Residual heterogeneity, because of remaining risk factors

We can recognize:

- \textit{observable} risk factors
  
  examples: health conditions, occupations, etc.

- \textit{non-observable} risk factors
  
  examples: individual’s attitude towards health, some congenital personal characteristics, etc.
RISK FACTORS IN ACTUARIAL PRACTICE

Observable risk factors

Usually accounted for via adjustment formulae, i.e. adjustments with respect to “normal” (or standard, or average) age-pattern of mortality

Examples, with reference to the force of mortality $\mu_x$

- Adjustments for *substandard risks*
  - linear model:
    $$\mu_x^{[A]} = a \mu_x + b, \text{ with } a > 1 \text{ and/or } b > 0$$

  in particular:

  \[
  \begin{align*}
  \mu_x^{[A]} &= a \mu_x \quad \text{multiplicative model} \\
  \mu_x^{[A]} &= \mu_x + b \quad \text{additive model}
  \end{align*}
  \]

  - age-shift model:
    $$\mu_x^{[A]} = \mu_{x+\tau}, \text{ with } \tau > 0$$
The age-pattern of mortality: heterogeneity issues  (cont’d)

- **Factor formula** (numerical rating system):

\[
\mu_{x}^{[\text{spec}]} = \left( 1 + \sum_{j=1}^{r} \gamma_{j} \right) \mu_{x} \quad \text{[i.e. \quad = a \, \mu_{x}, \text{ with } a > 0]} 
\]

- Adjustment for mortality of *disabled people*, allowing for disability past duration \(z\) (inception-select force of mortality) and disability category \(k\):

\[
\mu_{[x-z]+z}^{(k)} = a_{z}^{(k)} \mu_{x} + b_{z}^{(k)} \quad \text{with} \quad a_{z}^{(k)} > 1 \quad \text{and/or} \quad b_{z}^{(k)} > 0
\]

Adjustments frequently applied to \(q_{x}\) (rather than \(\mu_{x}\)), or directly to premiums (approx proportional to probabilities of dying in term insurance products)
The age-pattern of mortality: heterogeneity issues  (cont’d)

Non-observable risk factors

Actually disregarded

The impact of non-observable risk factors on the probability distribution of the random number of deaths should be assessed

An important result (see e.g. Pollard [1970]):

(a) if heterogeneity in a population is known in terms of probabilities and size of homogeneous subgroups ⇒ variance of the random number of deaths lower than in homogeneous population

(b) if heterogeneity in a population is random, in terms of probabilities and size of homogeneous subgroups ⇒ variance of the random number of deaths higher than in homogeneous population

Non-observable risk factors ⇒ case (b)
3  MODELING UNOBSERVABLE CAUSES
OF HETEROGENEITY

APPROACHES

Various approaches proposed to quantify the heterogeneity due to
unobservable risk factors

*The fixed frailty approach*

Heterogeneity described by individual *frailty*: a non-negative,
real-valued random variable which affects the individual force of
mortality

Individual frailty is unknown, but assumed constant lifelong

Approach proposed by Beard [1959, 1971] and Vaupel et al. [1979],
followed by numerous contributions; see in particular: Hougaard [1984,
1986], Manton et al. [1986], Steinsaltz and Wachter [2006],
Yashin et al. [1985], Yashin and Iachine [1997]

Applications to life annuities: for example, Butt and Haberman [2004],
Olivieri [2006]

Compact review provided by Haberman and Olivieri [2014]
The changing frailty approach
Model based on individual frailty stochastically changing with age proposed by Le Bras [1976]
Fixed frailty approach and changing frailty approach compared by Thatcher [1999] and Yashin et al. [1994]

The frailty-discrete approach
Basic idea: to split a heterogeneous population (a cohort, in particular) into a given number of homogeneous groups, each group characterized by a given age-pattern of mortality
Contributions provided in particular by Keyfitz and Littman [1979], Levinson [1959] and Redington [1969]
Detailed review provided by Olivieri [2006]
Among the most recent contributions: Avraam et al. [2014]
MORE ON THE FIXED FRAILTY APPROACH

Underlying idea: people with a higher frailty die on average earlier than others

The context:

- refer to a cohort, defined at age 0 and closed to new entrants
- consider people current age $x \Rightarrow$ heterogeneous group, because of unobservable factors
- assume that, for any individual, such factors are summarized by a non-negative variable, viz the frailty
- the specific value of the frailty of each individual does not change over time, but remains unknown
- because of deaths, the distribution of people in respect of frailty does change with age, as people with low frailty are expected to live longer
Formally:

- $Z_x = \text{random frailty at age } x$
- $\text{continuous probability distribution of frailty at age } x$, with pdf $g_x(z)$
- $\mu_x(z) = \text{(conditional) force of mortality of an individual current age } x \text{ with frailty level } z$

$$\mu_x(z) = \lim_{t \to 0} \frac{P[T_x \leq t | Z_x = z]}{t}$$

- $\mu_x = \mu_x(1) = \text{standard force of mortality}$

Assumption (multiplicative link, see Vaupel et al. [1979]):

$$\mu_x(z) = z \mu_x$$
Modeling unobservable causes of heterogeneity  (cont’d)

It can be proved (see e.g. Pitacco et al. [2009]) that, given

- the pdf of the initial distribution of the frailty, \( g_0(z) \)
- the (individual) force of mortality \( \mu_x(z) \) for \( x, z > 0 \)

we can determine:

- the pdf of the frailty at age \( x \), \( g_x(z) \), for \( x > 0 \);
- the average force of mortality in the cohort \( \bar{\mu}_x \) which, according to the multiplicative link, is given by:

\[
\bar{\mu}_x = \mu_x \bar{z}_x
\]

where \( \bar{z}_x \) is the expected frailty at age \( x \)

Choices needed to further progress in analytical terms:

- pdf of the frailty at a given age, e.g. age 0 \( \Rightarrow g_0(z) \)
- mortality law, i.e. a specific parametric model for \( \mu_x \)
Modeling unobservable causes of heterogeneity  

According to Beard [1959] and Vaupel et al. [1979]:

1. multiplicative model to link the frailty-specific force of mortality to the standard one

2. probability distribution of the frailty described by a Gamma with given parameters, \( \text{Gamma}(\delta, \theta) \)

3. Gompertz law or Makeham law for standard mortality; in what follows, we adopt the Gompertz law \( \mu_x = \alpha e^{\beta x} \Rightarrow \text{Gompertz - Gamma model} \)

We then find:

\[
\bar{\mu}_x = \frac{\alpha' e^{\beta x}}{\delta' e^{\beta x} + 1}
\]

i.e. Beard law, with \( \alpha', \delta' \) depending on \( \delta, \theta \)
Beard law:

- belongs to the *logistic class*
  - other laws proposed by Perks [1932], Kannisto [1994], Thatcher [1999] (see e.g. Pitacco [2016])
- coincides with the 1st Perks law, with $\gamma = 0$

Laws belonging to the logistic class $\Rightarrow$ *deceleration* in late-life mortality

Hence: Gompertz - Gamma model $\Rightarrow$ Beard law $\Rightarrow$ deceleration implied by individual frailty in a cohort
THE “DECELERATION” IN LATE-LIFE MORTALITY

Controversial issue, conflicting results from statistical data

See, for example: Thatcher [1999], Horiuchi and Wilmoth [1998], Gavrilov and Gavrilova [2011], CMI Working Paper 85 [2015]; for a short review, see Pitacco [2016]

In traditional parametric models (Gompertz, Makeham and Thiele)
force of mortality increases exponentially (at least definitely)
⇒ constant rate of increase

“Deceleration” phenomenon occurs when the force of mortality eventually increases at a decreasing rate

Generally speaking. i.e. not restricting the context to mortality of humans, the following mortality profiles, which decelerate at high ages, can be recognized
The force of mortality increases at a decreasing rate, for example because it eventually follows a linear profile (or approaches a slant linear asymptote).

The force of mortality stops increasing (or tends to a horizontal asymptote), and then proceeds at a constant rate (or approximately constant rate); hence, the rate of increase is (or tends to be) equal to zero ⇒ a mortality leveling-off occurs (or a mortality plateau is reached).

In some species, the force of mortality can eventually decline at old ages ⇒ negative rate of increase (meaning of “old” being of course related to the species addressed).

See following Figure
Modeling unobservable causes of heterogeneity (cont’d)

-force of mortality $\mu_x$

 ESPONENTIALLY INCREASING

(possible) slant linear asymptote

Leveling - off

Declining

DECELERATING

Force of mortality: exponentially increasing vs decelerating
4 LIFE ANNUITY PORTFOLIOS: FROM HOMOGENEITY TO HETEROGENEITY

We refer to portfolios of immediate life annuities (briefly, annuities), with annual benefits payable in arrears. We assume that the single premium is calculated according to the equivalence principle, relying on:

- interest rate
- life table
  - usually a projected life table, allowing for self-selection, for standard annuities
  - adjusted according to the result of underwriting process for special-rate annuities (or “underwritten” annuities); see Ridsdale [2012], Pitacco [2014]

Consider the following (actual or hypothetical) portfolio structures...
Common situation, due to strong self-selection effect: only people in good health conditions purchase a (standard) life annuity.

Result:

- size of the annuity portfolio small w.r.t. the number of potential clients (propensity to annuitize should also be considered)
- probably, very low degree of heterogeneity
Larger and more heterogeneous portfolio, hence:

- larger size $\Rightarrow$ contributes to lower variance in portfolio results (as regards the risk of random fluctuations)
- higher heterogeneity, because of non-observable risk factors $\Rightarrow$ contributes to raise variance in portfolio results
- what about the trade-off?

Unrealistic situation: a lower annuity price would be required to attract more clients and enlarge the annuity portfolio
A more realistic situation: the portfolio consists of

- standard annuities
- special-rate annuities (or underwritten annuities), sold to people in non-optimal health conditions
Larger and heterogeneous portfolio, hence:

- larger size $\Rightarrow$ contributes to lower variance in portfolio results (as regards the risk of random fluctuations)

- heterogeneity in the combined portfolio $\Rightarrow$ contributes to raise variance in portfolio results
  - heterogeneity among sub-portfolios
  - some degree of residual heterogeneity inside each sub-portfolio, because of residual non-observable risk factors (the underwriting process only provides a proxy)

- what about the trade-off?

Heterogeneity and related impact on life annuities, in terms of pricing, reserving, capital allocation, etc.: a new field of research; see e.g. Meyricke and Sherris [2013], Sherris and Zhou [2014], Su and Sherris [2012]
RISK CLASSIFICATION BASED ON A FRAILTY MODEL

Aim: to split a heterogeneous population into classes (groups) of individuals with similar risk profile ⇒ each class with reduced heterogeneity (w.r.t. heterogeneity in the population)

Identification of risk classes

Assume the individual frailty as the classification driver
Assess approximately the individual frailty via medical examination (possible step of the underwriting process)
Let \((z_{j-1}, z_j]\) denote the range of frailty values for risk group \(G_j\)
At age \(x\), \(G_j\) is defined as follows:

\[ G_j = \{i : z_{j-1} < Z_x^{(i)} \leq z_j\} \]

with \(Z_x^{(i)} = \text{frailty of individual } i\)
Let $J$ denote the number of groups

Assume $z_0 = 0, z_J \to \infty$

$\Rightarrow$ set $\{G_j; j = 1, \ldots J\}$ constitutes a partition of the sample space of frailty

**Lifetime and frailty in the risk classes**

Next step: to derive the probability distribution of the frailty for each group $G_j$, and the main summary statistics

The probability distribution of the frailty for group $G_j$ can be assessed as a conditional distribution of the frailty for the whole population

Let

- $\theta(x)$ denote the parameter of the Gamma distribution of $Z_x$
- $F(z; \delta, \theta(x))$ denote the probability distribution function of a Gamma($\delta, \theta(x)$)
Heterogeneity and portfolio size: a possible trade-off? (cont’d)

Relative size of group $G_j$:

$$\rho_{j;x} = \mathbb{P}[z_{j-1} < Z_x \leq z_j] = F(z_j; \delta, \theta(x)) - F(z_{j-1}; \delta, \theta(x))$$

with $\sum_{j=1}^{J} \rho_{j;x} = 1$

Probability distribution function of the frailty in group $G_j$ at age $x$:

$$F(z; \delta, \theta(x) \mid G_j) = \begin{cases} 
0 & \text{if } z \leq z_{j-1} \\
\frac{F(z; \delta, \theta(x)) - F(z_{j-1}; \delta, \theta(x))}{\rho_{j;x}} & \text{if } z_{j-1} < z \leq z_j \\
1 & \text{if } z > z_j 
\end{cases}$$

with expected value:

$$\mathbb{E}[Z_x \mid G_j] = \mathbb{E}[Z_x] \frac{F(z_j; \delta + 1, \theta(x)) - F(z_{j-1}; \delta + 1, \theta(x))}{\rho_{j;x}}$$
Heterogeneity and portfolio size: a possible trade-off? (cont’d)

and variance:

$$\text{Var}[Z_x | G_j] = \left( (\delta + 1) \frac{F(z_j; \delta + 2, \theta(x)) - F(z_{j-1}; \delta + 2, \theta(x))}{\rho_j; x} \right. $$

\[ - \delta \left( \frac{F(z_j; \delta + 1, \theta(x)) - F(z_{j-1}; \delta + 1, \theta(x))}{\rho_j; x} \right) \]

Average survival function in group $G_j$ at age $x$ can be derived from:

$$\tilde{S}(x | G_j) = \int_{z_{j-1}}^{z_j} S(x | z) g_0(z) \frac{1}{\rho_j; x} \, dz$$
MODEL CALIBRATION

Model described above applied to immediate life annuities
Refer to a cohort of males, initial age $x_0 = 65$

Assume that:
- $G_1$ collects standard risks
- other possible groups ($G_2$, $G_3$, ...) collect substandard days

Gompertz-Gamma model calibrated on Italian projected life tables, in particular:
- TG62 (general population - source: ISTAT)
- A62I (voluntary immediate life annuities - source: ANIA)

For a detailed presentation of the procedure, see Olivieri and Pitacco [2016]
Heterogeneity and portfolio size: a possible trade-off? (cont’d)

| Group | Frailty interval $\rho_{j,65}$ | Relative size at age 65 of group $G_j$ in the general population $\rho_{j,65}$ | Expected value of the frailty $\mathbb{E}[Z_{65}|G_j]$ | Coefficient of variation $\text{CV}[Z_{65}|G_j]$ | Expected lifetime $\mathbb{E}[T_{65}|G_j]$ |
|-------|-------------------------------|-----------------------------------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $G_1$ | $(0, 1.038741]$               | 60.121%                                                   | 0.845593                        | 15.243%                         | 22.81                           |
| $G_2$ | $(1.038741, 1.307144]$         | 30.111%                                                   | 1.152338                        | 6.479%                          | 20.36                           |
| $G_3$ | $(1.307144, \infty )$         | 9.769%                                                    | 1.445866                        | 8.736%                          | 18.71                           |
| Population | $(0, \infty )$              | 100%                                                      | 0.996594                        | 23.308%                         | 21.67                           |

*Risk classes*
Heterogeneity and portfolio size: a possible trade-off? (cont’d)

THE VALUE OF LIABILITIES OF A LIFE ANNUITY PORTFOLIO

Present value of benefits

Annual benefit for each risk in group $G_j$, according to the equivalence principle:

$$b_j = II \frac{1}{a_{x_0;j}}$$

where $II$ is the single premium (amount annuitized), and

$$a_{x_0;j} = \sum_{s=1}^{\infty} (1 + r)^{-s} \frac{S(x_0 + s | G_j)}{S(x_0 | G_j)}$$

Number of individuals in group $G_j$:

- $n_{0;j}$ (known) at time 0
- $N_{t;j}$ (random) at time $t$
Heterogeneity and portfolio size: a possible trade-off? (cont’d)

Present value at time $t$ of future benefits for group $G_j$:

$$PV_{t;j} = \sum_{s=t+1}^{\infty} b_j N_{t;j} v(t, s)$$

with $v(t, s) = \text{discount factor}$, assumed deterministic, in particular

$$v(t, s) = (1 + r)^{-(s-t)}$$

For the whole portfolio:

$$PV_t = \sum_{j=1}^{J} PV_{t;j}$$

To analyze the value of portfolio liabilities, we assess in particular:

- expected values $\mathbb{E}[PV_t]$
- coefficients of variation $\mathbb{CV}[PV_t]$
- right tails, through $\varepsilon$-percentiles $PV_{t[\varepsilon]}$, with e.g. $\varepsilon = 0.99$
Remark

Note that:

- according to the multiplicative model for adjusting the age-pattern of mortality, we have $\mu_x^{[A]} = a \mu_x$
  - a fixed value of $a$ is chosen for each risk, relying on medical examination
  - annuity benefit is calculated according to the related survival function
  - heterogeneity in the group with a given $a$ is not explicitly accounted for

- according to the frailty multiplicative link, we have $\mu_x(z) = z \mu_x$
  - a group is defined by the related interval $(z_{j-1}, z_j]$ of frailty values
  - annuity benefit for group $G_j$ is calculated according to the average survival function in that group, $\bar{S}(x \mid G_j)$
  - present value at time $t$ of benefits in group $G_j$, $PV_{t;j}$, is a random quantity, whose volatility, also due to residual heterogeneity inside the group, can be assessed
Heterogeneity and portfolio size: a possible trade-off?

(cont’d)

Numerical investigation

We consider six alternative portfolios (see Table):

- portfolios A - E differ for the size of groups $G_2$ and $G_3$, and possibly the total portfolio size
- portfolio F has the same size of A, but a different composition

<table>
<thead>
<tr>
<th>Groups</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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</thead>
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<td>$G_1$</td>
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<td>500</td>
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<td>1250</td>
<td>1250</td>
<td>1663</td>
<td>1000</td>
</tr>
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</table>

Alternative portfolios
Heterogeneity and portfolio size: a possible trade-off? (cont’d)

We set \( n_{0;1} = 1000 \). According to the frailty model:

- maximum size at time \( t = 0 \) of the portfolio \( \Rightarrow \frac{1000}{\rho_{1;65}} = 1663 \)
- maximum size at time \( t = 0 \) of group \( G_2 \) \( \Rightarrow 1663 \rho_{2;65} = 501 \)
- maximum size at time \( t = 0 \) of group \( G_3 \) \( \Rightarrow 1663 \rho_{3;65} = 162 \)

In particular, note that:

- portfolio A is the base case; it only consists of standard risks
- portfolio E has the largest possible size, including policies in groups \( G_2 \) and \( G_3 \)
- portfolio B and C include some policies in group \( G_2 \), where C has a larger size
- portfolio D has the same size as portfolio C, but with some policies also in group \( G_3 \)
- portfolio F shows adverse-selection: the number of standard risks is lower than in the other cases, but the size is the same as portfolio A, due to risks in class \( G_2 \)
Heterogeneity and portfolio size: a possible trade-off? (cont’d)

<table>
<thead>
<tr>
<th>Time $t$</th>
<th>Portfolio A</th>
<th>Portfolio B</th>
<th>Portfolio C</th>
<th>Portfolio D</th>
<th>Portfolio E</th>
<th>Portfolio F</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Abs. value</td>
<td>% of the value obtained for Portfolio A</td>
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</tr>
</tbody>
</table>

*Expected present value of future benefits, per policy in-force:* \( \frac{\mathbb{E}[PV_t]}{n_t} \)
Heterogeneity and portfolio size: a possible trade-off? (cont’d)

<table>
<thead>
<tr>
<th>Time $t$</th>
<th>Portfolio A</th>
<th>Portfolio B</th>
<th>Portfolio C</th>
<th>Portfolio D</th>
<th>Portfolio E</th>
<th>Portfolio F</th>
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</table>

Coefficient of variation of the present value of future benefits: $\text{CV}[PV_t]$
Heterogeneity and portfolio size: a possible trade-off? (cont’d)

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<th>Time ( t )</th>
<th>Portfolio A</th>
<th>Portfolio B</th>
<th>Portfolio C</th>
<th>Portfolio D</th>
<th>Portfolio E</th>
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</table>

99th percentile of \( PV_t \), as a % of \( \mathbb{E}[PV_t] \):

\[
\frac{PV_{t[0.99]}}{\mathbb{E}[PV_t]}\]
Main findings and related interpretations

We focus on portfolios risk profile, and its relation with heterogeneity degree

Looking at coefficients of variation and/or percentiles we note what follows

- portfolio F: the highest riskiness
- comparing F to A: same size, but in F more heterogeneity (groups $G_1$ and $G_2$) not counterbalanced by larger size $\Rightarrow$ higher riskiness
- portfolio E: high heterogeneity (groups $G_1$, $G_2$ and $G_3$) counterbalanced by the largest size $\Rightarrow$ lowest riskiness (even lower than portfolio A, thanks to larger size)

- Higher degrees of heterogeneity $\Rightarrow$ higher risk profile
- If matched by larger total portfolio size, risk profile can benefit from portfolio diversification
6 CONCLUDING REMARKS

To summarize:

- Heterogeneity in age-pattern of mortality
  - Observable risk factors
  - Unobservable risk factors
- Fixed-frailty models
- Changing-frailty models
- Discrete models
- Risk classification
- Deceleration
- Application to life annuity portfolios
Concluding remarks (cont’d)

From an ERM perspective:

- **risk identification** ⇒ awareness of heterogeneity and unobservable risk factors
- **risk assessment** ⇒ biometric assumptions, i.e. mortality law and frailty model
- **impact assessment** ⇒ probability distribution of the PV of portfolio liabilities, and related synthetic values (risk measures in particular)
- **RM actions** ⇒ risk classification and product design and pricing

Don’t forget: the first phase of the RM process is *objective setting*; if
- raising the market share
- creating clients’ value

are among the objectives, then product design and related appropriate pricing are the most critical issues
Basic references

Where links are provided, they were active as of the time this presentation was completed but may have been updated since then.


Basic references (cont’d)


http://www.actuaweb.be/frameset/frameset.html


E. Pitacco. High age mortality and frailty. some remarks and hints for actuarial modeling. CEPAR Working Paper 2016/19, 2016. Available at: 


Basic references (cont’d)


Many thanks for your kind attention!