Heterogeneity in a life annuity portfolio: modeling issues and risk profile assessment

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Agenda

1. Introduction & motivation
2. The age-patterns of mortality: heterogeneity issues
3. Modeling unobservable causes of heterogeneity
4. Life annuity portfolios: from homogeneity to heterogeneity
5. Heterogeneity and portfolio size: a possible trade-off ?
6. Concluding remarks

Presentation based on:

- ▷ *joint work with Annamaria Olivieri, University of Parma*
- ▷ *research work organized by the Mortality Working Group, International Actuarial Association*

1 INTRODUCTION & MOTIVATION

Heterogeneity in an insured population: a classical topic in non-life actuarial mathematics

Models for observable risk factors (in particular rating factors) and non-observable risk factors

Non-observable risk factors: classical Poisson - Gamma model for the random number of claims

In life actuarial mathematics, focus on heterogeneity due to non-observable risk factors: first contribution by Beard [1959] (unfortunately an “obscure” paper, for many years unknown to actuaries !)

More contributions in demography; in particular, starting from Vaupel et al. [1979]

Recently, interest in application to life insurance field, in particular life annuities; for an extensive list of references, see Pitacco [2016]

Aims of this talk:

- ▷ to recall basic ideas about heterogeneity in age-pattern of mortality, due to observable and non-observable causes, and to focus on relevant modeling issues (Sects. 2 and 3)

  For more info, see Pitacco [2016]

- ▷ to address some features of life annuity portfolios, looking for possible risk classification criteria and consequent differentiated pricing (Sects. 4 and 5)

  See the paper by Olivieri and Pitacco [2016]

2 THE AGE-PATTERN OF MORTALITY: HETEROGENEITY ISSUES

CAUSES OF HETEROGENEITY IN A POPULATION

For each individual in the population:

- ▷ age
- ▷ gender
- ▷ health conditions
- ▷ occupation
- ▷ genetic factors
- ▷ environmental factors
- ▷ ... ...

$\Rightarrow$ *risk factors* in the actuarial language (non-necessarily *rating factors*, because of legislation, market features, etc.)

Refer to a cohort, given gender

Residual heterogeneity, because of remaining risk factors

We can recognize:

- *observable* risk factors

  examples: health conditions, occupations, etc.

- *non-observable* risk factors

  examples: individual's attitude towards health, some congenital personal characteristics, etc.

RISK FACTORS IN ACTUARIAL PRACTICE

Observable risk factors

Usually accounted for via adjustment formulae, i.e. adjustments with respect to "normal" (or standard, or average) age-pattern of mortality

Examples, with reference to the force of mortality μ_x

- Adjustments for *substandard risks*
  - ▷ linear model:

$$\mu_x^{[A]} = a\,\mu_x + b, \text{ with } a > 1 \text{ and/or } b > 0$$

in particular:

$$\underbrace{\mu_x^{[A]} = a\,\mu_x}_{\text{multiplicative model}} \quad \text{or} \quad \underbrace{\mu_x^{[A]} = \mu_x + b}_{\text{additive model}}$$

  - ▷ age-shift model:

$$\mu_x^{[A]} = \mu_{x+\tau}, \text{ with } \tau > 0$$

- *Factor formula* (numerical rating system):

$$\mu_x^{[\text{spec}]} = \Big(1 + \sum_{j=1}^{r} \gamma_j\Big)\mu_x \qquad \Big[\text{i.e.} \;\; = a\,\mu_x, \text{ with } \; a > 0\Big]$$

- Adjustment for mortality of *disabled people*, allowing for disability past duration z (inception-select force of mortality) and disability category k:

$$\mu_{[x-z]+z}^{(k)} = a_z^{(k)}\,\mu_x + b_z^{(k)}, \;\; \text{with } \; a_z^{(k)} > 1 \; \text{ and/or } \; b_z^{(k)} > 0$$

Adjustments frequently applied to q_x (rather than μ_x), or directly to premiums (approx proportional to probabilities of dying in term insurance products)

Non-observable risk factors

Actually disregarded

The impact of non-observable risk factors on the probability distribution of the random number of deaths should be assessed

An important result (see e.g. Pollard [1970]):

(a) if heterogeneity in a population is *known* in terms of probabilities and size of homogeneous subgroups $\Rightarrow$ variance of the random number of deaths lower than in homogeneous population

(b) if heterogeneity in a population is *random*, in terms of probabilities and size of homogeneous subgroups $\Rightarrow$ variance of the random number of deaths higher than in homogeneous population

Non-observable risk factors $\Rightarrow$ case (b)

3 MODELING UNOBSERVABLE CAUSES OF HETEROGENEITY

APPROACHES

Various approaches proposed to quantify the heterogeneity due to unobservable risk factors

The fixed frailty approach

Heterogeneity described by individual *frailty*: a non-negative, real-valued random variable which affects the individual force of mortality

Individual frailty is unknown, but assumed constant lifelong

Approach proposed by Beard [1959, 1971] and Vaupel et al. [1979], followed by numerous contributions; see in particular: Hougaard [1984, 1986], Manton et al. [1986], Steinsaltz and Wachter [2006], Yashin et al. [1985], Yashin and Iachine [1997]

Applications to life annuities: for example, Butt and Haberman [2004], Olivieri [2006]

Compact review provided by Haberman and Olivieri [2014]

The changing frailty approach

Model based on individual frailty stochastically changing with age proposed by Le Bras [1976]

Fixed frailty approach and changing frailty approach compared by Thatcher [1999] and Yashin et al. [1994]

Markov aging models, which generalize Le Bras's assumption, adopted by Su and Sherris [2012], Lin and Liu [2007], Liu and Lin [2012] and Sherris and Zhou [2014]

The frailty-discrete approach

Basic idea: to split a heterogeneous population (a cohort, in particular) into a given number of homogeneous groups, each group characterized by a given age-pattern of mortality

Contributions provided in particular by Keyfitz and Littman [1979], Levinson [1959] and Redington [1969]

Detailed review provided by Olivieri [2006]

Among the most recent contributions: Avraam et al. [2014]

More on the fixed frailty approach

Underlying idea: people with a higher frailty die on average earlier than others

The context:

- refer to a cohort, defined at age 0 and closed to new entrants
- consider people current age x $\Rightarrow$ heterogeneous group, because of unobservable factors
- assume that, for any individual, such factors are summarized by a non-negative variable, viz the frailty
- the specific value of the frailty of each individual does not change over time, but remains unknown
- because of deaths, the distribution of people in respect of frailty does change with age, as people with low frailty are expected to live longer

Formally:

- ▷ Z_x = random frailty at age x
- ▷ continuous probability distribution of frailty at age x, with pdf $g_x(z)$
- ▷ $\mu_x(z)$ = (conditional) force of mortality of an individual current age x with frailty level z

$$\mu_x(z) = \lim_{t \to 0} \frac{\mathbb{P}[T_x \leq t | Z_x = z]}{t}$$

- ▷ $\mu_x = \mu_x(1)$ = standard force of mortality

Assumption (multiplicative link, see Vaupel et al. [1979]):

$$\mu_x(z) = z\, \mu_x$$

It can be proved (see e.g. Pitacco et al. [2009]) that, given

- ▷ the pdf of the initial distribution of the frailty, $g_0(z)$
- ▷ the (individual) force of mortality $\mu_x(z)$ for $x, z > 0$

we can determine:

- the pdf of the frailty at age x, $g_x(z)$, for $x > 0$;
- the average force of mortality in the cohort $\bar{\mu}_x$ which, according to the multiplicative link, is given by:

$$\bar{\mu}_x = \mu_x \, \bar{z}_x$$

  where $\bar{z}_x$ is the expected frailty at age x

Choices needed to further progress in analytical terms:

- ▷ pdf of the frailty at a given age, e.g. age $0 \Rightarrow g_0(z)$
- ▷ mortality law, i.e. a specific parametric model for μ_x

According to Beard [1959] and Vaupel et al. [1979]:

(1) multiplicative model to link the frailty-specific force of mortality to the standard one

(2) probability distribution of the frailty described by a Gamma with given parameters, $\mathrm{Gamma}(\delta, \theta)$

(3) Gompertz law or Makeham law for standard mortality; in what follows, we adopt the Gompertz law $\mu_x = \alpha\, \mathrm{e}^{\beta\, x} \;\Rightarrow$ *Gompertz - Gamma model*

We then find:

$$\bar{\mu}_x = \frac{\alpha'\, \mathrm{e}^{\beta x}}{\delta'\, \mathrm{e}^{\beta x} + 1}$$

i.e. Beard law, with α', δ' depending on δ, θ

Beard law:

- belongs to the *logistic class*
  - ▷ other laws proposed by Perks [1932], Kannisto [1994], Thatcher [1999] (see e.g. Pitacco [2016])
- coincides with the 1st Perks law, with $\gamma = 0$

Laws belonging to the logistic class $\Rightarrow$ *deceleration* in late-life mortality

Hence: Gompertz - Gamma model $\Rightarrow$ Beard law $\Rightarrow$ deceleration implied by individual frailty in a cohort

THE "DECELERATION" IN LATE-LIFE MORTALITY

Controversial issue, conflicting results from statistical data

See, for example: Thatcher [1999], Horiuchi and Wilmoth [1998], Gavrilov and Gavrilova [2011], CMI Working Paper 85 [2015]; for a short review, see Pitacco [2016]

In traditional parametric models (Gompertz, Makeham and Thiele) force of mortality increases exponentially (at least definitely)
$\Rightarrow$ constant rate of increase

"Deceleration" phenomenon occurs when the force of mortality eventually increases at a decreasing rate

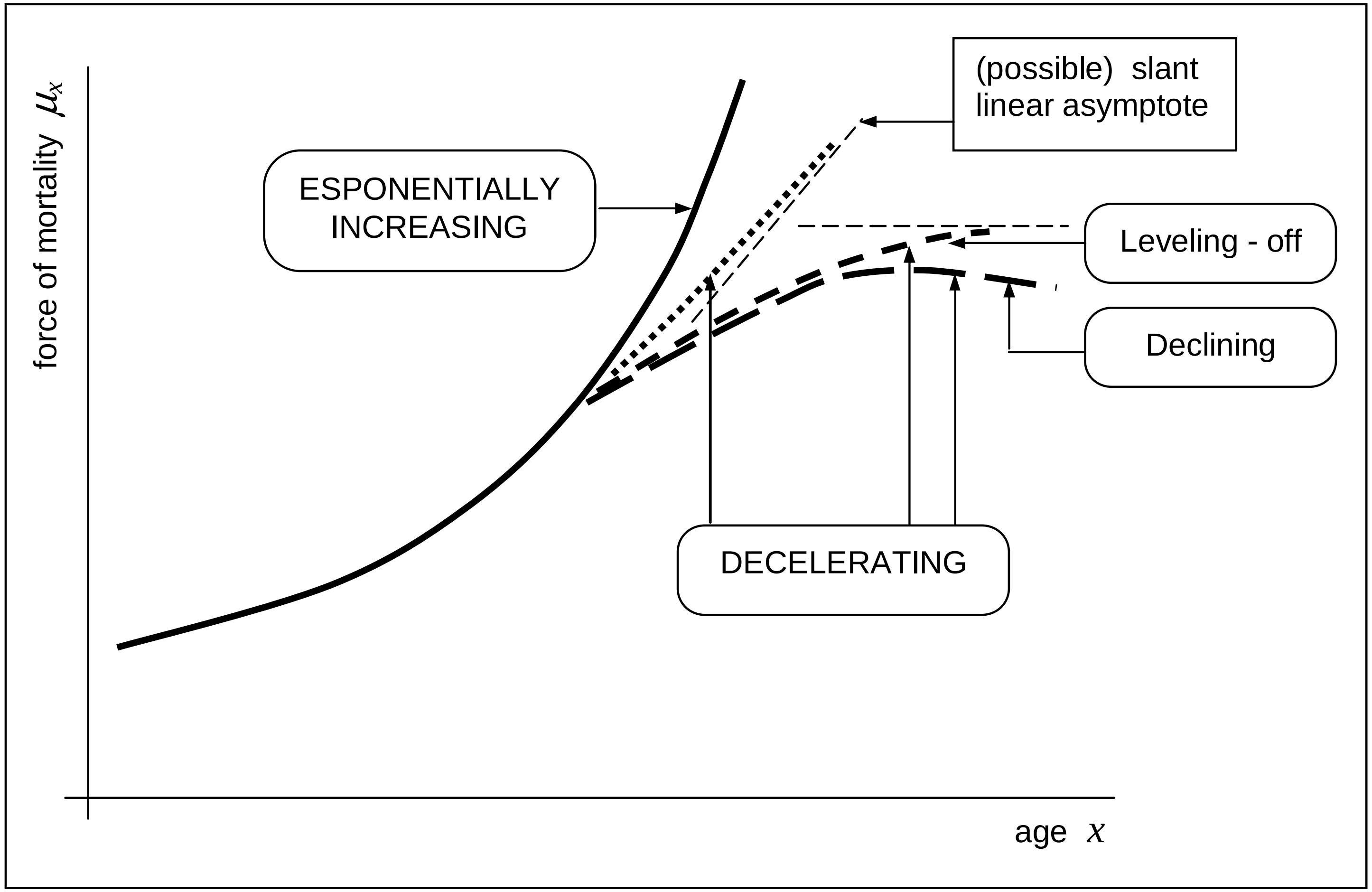
Generally speaking. i.e. not restricting the context to mortality of humans, the following mortality profiles, which decelerate at high ages, can be recognized

Modeling unobservable causes of heterogeneity *(cont'd)*

- ▷ The *force of mortality increases at a decreasing rate*, for example because it eventually follows a linear profile (or approaches a slant linear asymptote)
- ▷ The *force of mortality stops increasing* (or tends to a horizontal asymptote), and then proceeds at a constant rate (or approximately constant rate); hence, the rate of increase is (or tends to be) equal to zero $\Rightarrow$ a mortality *leveling-off* occurs (or a mortality *plateau* is reached)
- ▷ In some species, the *force of mortality can eventually decline* at old ages $\Rightarrow$ negative rate of increase (meaning of "old" being of course related to the species addressed)

See following Figure

Modeling unobservable causes of heterogeneity *(cont'd)*



Force of mortality: exponentially increasing vs decelerating

4 LIFE ANNUITY PORTFOLIOS: FROM HOMOGENEITY TO HETEROGENEITY

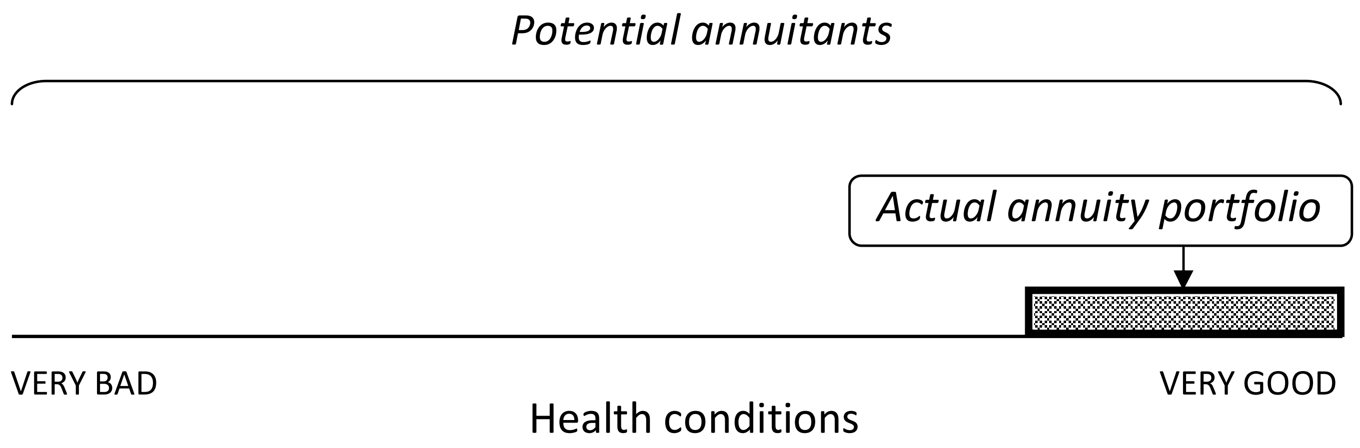
We refer to portfolios of immediate life annuities (briefly, annuities), with annual benefits payable in arrears

We assume that the single premium is calculated according to the equivalence principle, relying on:

- interest rate
- life table
  - ▷ usually a projected life table, allowing for self-selection, for standard annuities
  - ▷ adjusted according to the result of underwriting process for special-rate annuities (or “underwritten” annuities); see Ridsdale [2012], Pitacco [2014]

Consider the following (actual or hypothetical) portfolio structures

Life annuity portfolios: from homogeneity to heterogeneity *(cont'd)*

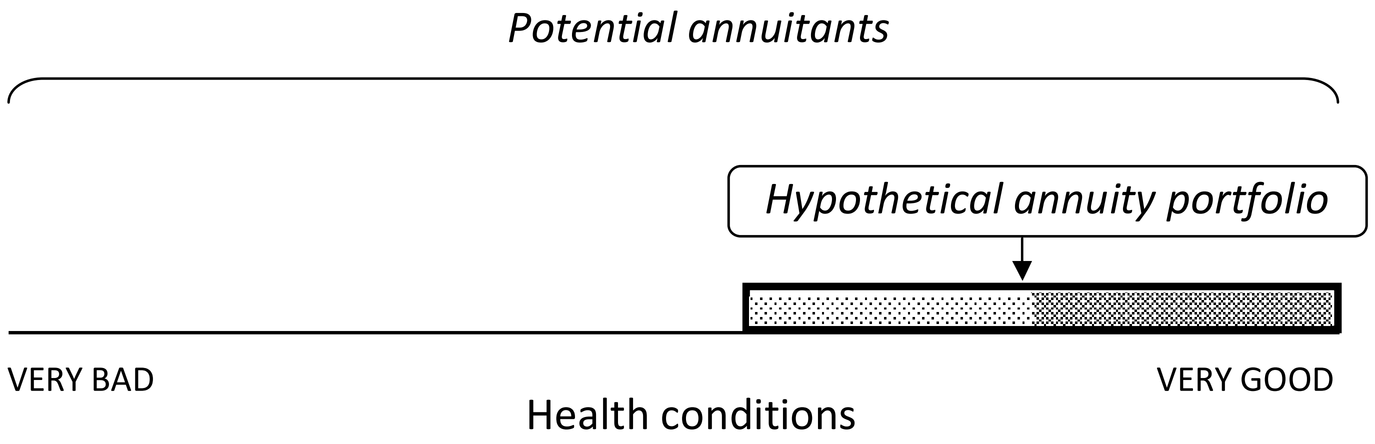


Common situation, due to strong self-selection effect: only people in good health conditions purchase a (standard) life annuity

Result:

- size of the annuity portfolio small w.r.t. the number of potential clients (propensity to annuitize should also be considered)
- probably, very low degree of heterogeneity

Life annuity portfolios: from homogeneity to heterogeneity *(cont'd)*

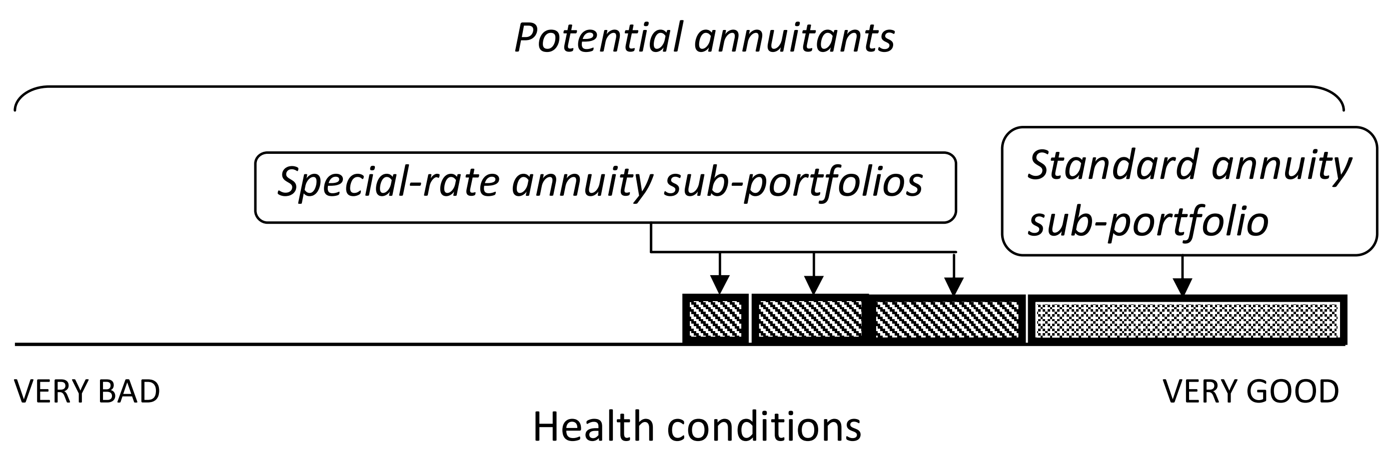


Larger and more heterogeneous portfolio, hence:

- larger size $\Rightarrow$ contributes to lower variance in portfolio results (as regards the risk of random fluctuations)
- higher heterogeneity, because of non-observable risk factors $\Rightarrow$ contributes to raise variance in portfolio results
- what about the trade-off ?

Unrealistic situation: a lower annuity price would be required to attract more clients and enlarge the annuity portfolio

Life annuity portfolios: from homogeneity to heterogeneity *(cont'd)*



A more realistic situation: the portfolio consists of

- ▷ standard annuities
- ▷ special-rate annuities (or underwritten annuities), sold to people in non-optimal health conditions

Life annuity portfolios: from homogeneity to heterogeneity *(cont'd)*

Larger and heterogeneous portfolio, hence:

- larger size $\Rightarrow$ contributes to lower variance in portfolio results (as regards the risk of random fluctuations)
- heterogeneity in the combined portfolio $\Rightarrow$ contributes to raise variance in portfolio results
  - ▷ heterogeneity among sub-portfolios
  - ▷ some degree of residual heterogeneity inside each sub-portfolio, because of residual non-observable risk factors (the underwriting process only provides a proxy)
- what about the trade-off ?

Heterogeneity and related impact on life annuities, in terms of pricing, reserving, capital allocation, etc.: a new field of research; see e.g. Meyricke and Sherris [2013], Sherris and Zhou [2014], Su and Sherris [2012]

5 HETEROGENEITY AND PORTFOLIO SIZE: A POSSIBLE TRADE-OFF ?

RISK CLASSIFICATION BASED ON A FRAILTY MODEL

Aim: to split a heterogeneous population into classes (groups) of individuals with similar risk profile $\Rightarrow$ each class with reduced heterogeneity (w.r.t. heterogeneity in the population)

Identification of risk classes

Assume the individual frailty as the classification driver

Assess approximately the individual frailty via medical examination (possible step of the underwriting process)

Let $(z_{j-1}, z_j]$ denote the range of frailty values for risk group G_j

At age x, G_j is defined as follows:

$$G_j = \{i : z_{j-1} < Z_x^{(i)} \leq z_j\}$$

with $Z_x^{(i)}$ = frailty of individual i

Let J denote the number of groups

Assume $z_0 = 0$, $z_J \to \infty$

$\Rightarrow$ set $\{G_j; j = 1, \dots J\}$ constitutes a partition of the sample space of frailty

Lifetime and frailty in the risk classes

Next step: to derive the probability distribution of the frailty for each group G_j, and the main summary statistics

The probability distribution of the frailty for group G_j can be assessed as a conditional distribution of the frailty for the whole population

Let

- $\theta(x)$ denote the parameter of the Gamma distribution of Z_x
- $F(z; \delta, \theta(x))$ denote the probability distribution function of a $\mathrm{Gamma}(\delta, \theta(x))$

Heterogeneity and portfolio size: a possible trade-off ? *(cont'd)*

Relative size of group G_j:

$$\rho_{j;x} = \mathbb{P}[z_{j-1} < Z_x \leq z_j] = F(z_j;\delta,\theta(x)) - F(z_{j-1};\delta,\theta(x))$$

with $\sum_{j=1}^{J} \rho_{j;x} = 1$

Probability distribution function of the frailty in group G_j at age x:

$$F(z;\delta,\theta(x)\,|\,G_j) = \begin{cases} 0 & \text{if } z \leq z_{j-1} \\ \dfrac{F(z;\delta,\theta(x)) - F(z_{j-1};\delta,\theta(x))}{\rho_{j;x}} & \text{if } z_{j-1} < z \leq z_j \\ 1 & \text{if } z > z_j \end{cases}$$

with expected value:

$$\mathbb{E}[Z_x\,|\,G_j] = \mathbb{E}[Z_x]\,\frac{F(z_j;\delta+1,\theta(x)) - F(z_{j-1};\delta+1,\theta(x))}{\rho_{j;x}}$$

and variance:

$$\mathbb{V}\mathrm{ar}[Z_x \,|\, G_j] = \Big((\delta+1)\,\frac{F(z_j;\delta+2,\theta(x)) - F(z_{j-1};\delta+2,\theta(x))}{\rho_{j;x}} \\ - \delta\,\frac{F(z_j;\delta+1,\theta(x)) - F(z_{j-1};\delta+1,\theta(x))}{\rho_{j;x}}\Big)$$

Average survival function in group G_j at age x can be derived from:

$$\bar{S}(x \,|\, G_j) = \int_{z_{j-1}}^{z_j} S(x \,|\, z)\, g_0(z)\, \frac{1}{\rho_{j;x}}\, \mathrm{d}z$$

MODEL CALIBRATION

Model described above applied to immediate life annuities

Refer to a cohort of males, initial age $x_0 = 65$

Assume that:

- G_1 collects standard risks
- other possible groups ($G_2, G_3, \dots$) collect substandard days

Gompertz-Gamma model calibrated on Italian projected life tables, in particular:

▷ TG62 (general population - source: ISTAT)

▷ A62I (voluntary immediate life annuities - source: ANIA)

For a detailed presentation of the procedure, see Olivieri and Pitacco [2016]

Heterogeneity and portfolio size: a possible trade-off ? *(cont'd)*

Group	**Frailty interval** $(z_{j-1}, z_j]$	**Relative size at age 65 of group G_j in the general population** $\rho_{j;65}$	**Expected value of the frailty** $\mathbb{E}[Z_{65}\|G_j]$	**Coefficient of variation** $\mathbb{CV}[Z_{65}\|G_j]$	**Expected lifetime** $\mathbb{E}[T_{65}\|G_j]$
G_1	$(0, 1.038741]$	60.121%	0.845593	15.243%	22.81
G_2	$(1.038741, 1.307144]$	30.111%	1.152338	6.479%	20.36
G_3	$(1.307144, \infty)$	9.769%	1.445866	8.736%	18.71
Population	$(0, \infty)$	100%	0.996594	23.308%	21.67

Risk classes

THE VALUE OF LIABILITIES OF A LIFE ANNUITY PORTFOLIO

Present value of benefits

Annual benefit for each risk in group G_j, according to the equivalence principle:

$$b_j = \mathit{\Pi} \, \frac{1}{a_{x_0;j}}$$

where $\mathit{\Pi}$ is the single premium (amount annuitized), and

$$a_{x_0;j} = \sum_{s=1}^{\infty} (1+r)^{-s} \, \frac{\bar{S}(x_0+s \,|\, G_j)}{\bar{S}(x_0 \,|\, G_j)}$$

Number of individuals in group G_j:

- ▷ $n_{0;j}$ (known) at time 0
- ▷ $N_{t;j}$ (random) at time t

Heterogeneity and portfolio size: a possible trade-off ? *(cont'd)*

Present value at time t of future benefits for group G_j:

$$PV_{t;j} = \sum_{s=t+1}^{\infty} b_j \, N_{t;j} \, v(t,s)$$

with $v(t,s)$ = discount factor, assumed deterministic, in particular $v(t,s) = (1+r)^{-(s-t)}$

For the whole portfolio:

$$PV_t = \sum_{j=1}^{J} PV_{t;j}$$

To analyze the value of portfolio liabilities, we assess in particular:

- expected values $\mathbb{E}[PV_t]$
- coefficients of variation $\mathbb{CV}[PV_t]$
- right tails, through ε-percentiles $PV_{t[\varepsilon]}$, with e.g. $\varepsilon = 0.99$

Heterogeneity and portfolio size: a possible trade-off ? *(cont'd)*

Remark

Note that:

- according to the multiplicative model for adjusting the age-pattern of mortality, we have $\mu_x^{[\mathrm{A}]} = a\,\mu_x$
  - ▷ a fixed value of a is chosen for each risk, relying on medical examination
  - ▷ annuity benefit is calculated according to the related survival function
  - ▷ heterogeneity in the group with a given a is not explicitly accounted for
- according to the frailty multiplicative link, we have $\mu_x(z) = z\,\mu_x$
  - ▷ a group is defined by the related interval $(z_{j-1}, z_j]$ of frailty values
  - ▷ annuity benefit for group G_j is calculated according to the average survival function in that group, $\bar{S}(x \mid G_j)$
  - ▷ present value at time t of benefits in group G_j, $PV_{t;j}$, is a random quantity, whose volatility, also due to residual heterogeneity inside the group, can be assessed

Numerical investigation

We consider six alternative portfolios (see Table):

- portfolios A - E differ for the size of groups G_2 and G_3, and possibly the total portfolio size
- portfolio F has the same size of A, but a different composition

Groups	**Portfolio**					
	A	**B**	**C**	**D**	**E**	**F**
G_1	1 000	1 000	1 000	1 000	1 000	500
G_2	0	200	250	200	501	500
G_3	0	0	0	50	162	0
All	1 000	1 200	1 250	1 250	1 663	1 000

Alternative portfolios

Heterogeneity and portfolio size: a possible trade-off ? *(cont'd)*

We set $n_{0;1} = 1\,000$. According to the frailty model:

- maximum size at time $t = 0$ of the portfolio $\Rightarrow \frac{1\,000}{\rho_{1;65}} = 1\,663$
- maximum size at time $t = 0$ of group $G_2 \Rightarrow 1\,663\,\rho_{2;65} = 501$
- maximum size at time $t = 0$ of group $G_3 \Rightarrow 1\,663\,\rho_{3;65} = 162$

In particular, note that:

- ▷ portfolio A is the base case; it only consists of standard risks
- ▷ portfolio E has the largest possible size, including policies in groups G_2 and G_3
- ▷ portfolio B and C include some policies in group G_2, where C has a larger size
- ▷ portfolio D has the same size as portfolio C, but with some policies also in group G_3
- ▷ portfolio F shows adverse-selection: the number of standard risks is lower than in the other cases, but the size is the same as portfolio A, due to risks in class G_2

Heterogeneity and portfolio size: a possible trade-off ? *(cont'd)*

Time t	Portfolio A	Portfolio B	Portfolio C	Portfolio D	Portfolio E	Portfolio F
	Abs. value	**% of the value obtained for Portfolio A**				
0	100.00	100.00%	100.00%	100.00%	100.01%	100.01%
5	81.26	99.71%	99.65%	99.60%	99.18%	99.13%
10	64.00	99.37%	99.24%	99.15%	98.24%	98.10%
15	48.62	99.00%	98.80%	98.66%	97.24%	96.94%
20	35.44	98.63%	98.35%	98.22%	96.25%	95.67%
25	24.66	98.32%	97.98%	97.89%	95.45%	94.47%
30	16.35	98.18%	97.77%	97.82%	95.13%	93.55%
35	10.34	98.31%	98.04%	97.94%	95.72%	93.71%
40	6.32	100.00%	100.00%	100.00%	97.63%	95.54%
45	3.93	100.00%	100.00%	100.00%	100.00%	0.00%

Expected present value of future benefits, per policy in-force: $\dfrac{\mathbb{E}[PV_t]}{n_t}$

Heterogeneity and portfolio size: a possible trade-off ? *(cont'd)*

Time t	Portfolio A	Portfolio B	Portfolio C	Portfolio D	Portfolio E	Portfolio F
0	1.30%	1.20%	1.17%	1.18%	1.04%	1.87%
5	1.48%	1.37%	1.34%	1.35%	1.19%	1.55%
10	1.75%	1.62%	1.60%	1.60%	1.39%	1.80%
15	2.10%	1.96%	1.91%	1.93%	1.70%	2.19%
20	2.64%	2.45%	2.41%	2.43%	2.17%	2.80%
25	3.55%	3.34%	3.31%	3.31%	3.04%	3.97%
30	5.62%	5.38%	5.32%	5.35%	4.96%	6.54%
35	11.10%	10.78%	10.78%	10.73%	10.28%	13.82%
40	32.19%	32.19%	32.19%	32.19%	31.40%	44.42%
45	136.25%	136.25%	136.25%	136.25%	136.25%	

Coefficient of variation of the present value of future benefits: $\mathbb{CV}[PV_t]$

Heterogeneity and portfolio size: a possible trade-off ? *(cont'd)*

Time t	Portfolio A	Portfolio B	Portfolio C	Portfolio D	Portfolio E	Portfolio F
0	103.07%	102.81%	102.70%	102.76%	102.44%	104.35%
5	103.46%	103.18%	103.06%	103.13%	102.73%	103.63%
10	104.12%	103.70%	103.77%	103.73%	103.22%	104.17%
15	104.98%	104.58%	104.48%	104.58%	104.00%	105.08%
20	106.36%	105.90%	105.73%	105.79%	105.15%	106.59%
25	108.28%	107.88%	107.78%	107.83%	106.98%	109.49%
30	113.66%	112.77%	112.77%	112.75%	111.91%	115.31%
35	126.74%	125.91%	125.69%	125.67%	124.79%	133.88%
40	185.49%	185.49%	185.49%	185.49%	181.62%	222.70%
45	570.00%	570.00%	570.00%	570.00%	570.00%	

99th percentile of PV_t, *as a % of* $\mathbb{E}[PV_t]$: $\dfrac{PV_{t[0.99]}}{\mathbb{E}[PV_t]}$

Main findings and related interpretations

We focus on portfolios risk profile, and its relation with heterogeneity degree

Looking at coefficients of variation and/or percentiles we note what follows

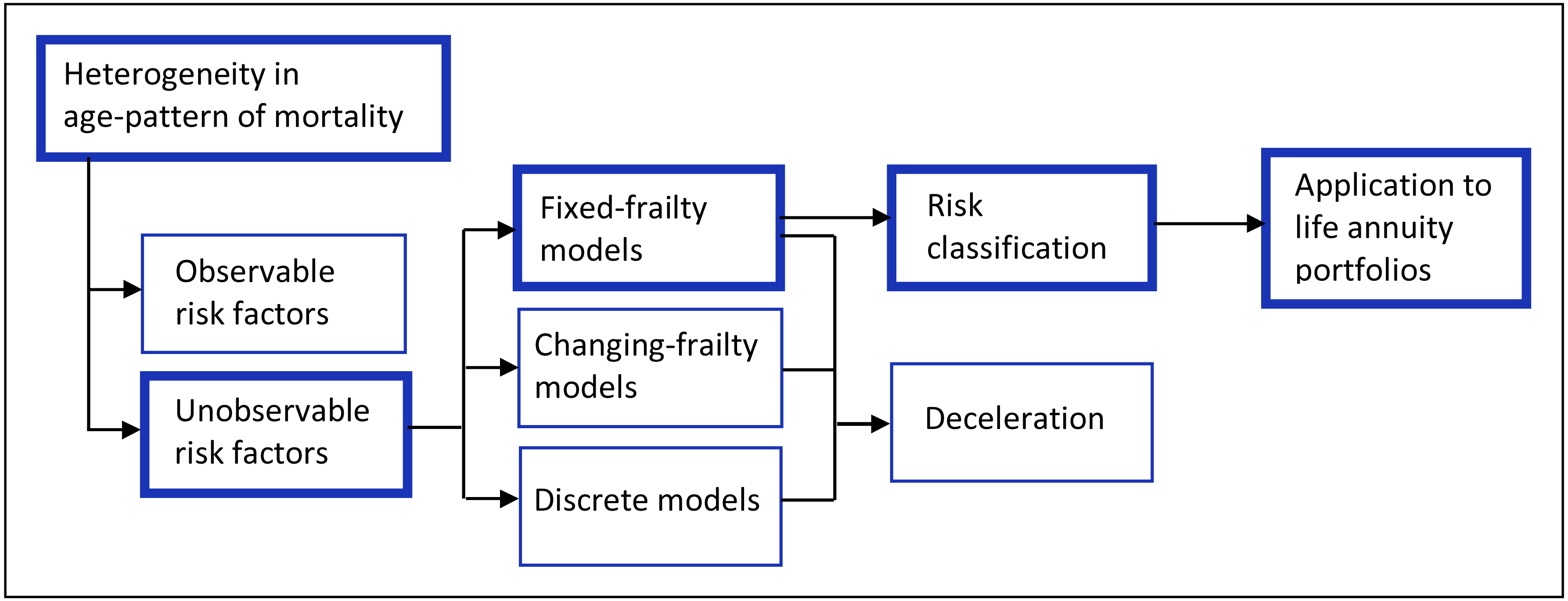
- portfolio F: the highest riskiness
- comparing F to A: same size, but in F more heterogeneity (groups G_1 and G_2) not counterbalanced by larger size $\Rightarrow$ higher riskiness
- portfolio E: high heterogeneity (groups G_1, G_2 and G_3) counterbalanced by the largest size $\Rightarrow$ lowest riskiness (even lower than portfolio A, thanks to larger size)

▷ Higher degrees of heterogeneity $\Rightarrow$ higher risk profile

▷ If matched by larger total portfolio size, risk profile can benefit from portfolio diversification

6 CONCLUDING REMARKS

To summarize:



From an ERM perspective:

- ▷ *risk identification* ⇒ awareness of heterogeneity and unobservable risk factors
- ▷ *risk assessment* ⇒ biometric assumptions, i.e. mortality law and frailty model
- ▷ *impact assessment* ⇒ probability distribution of the PV of portfolio liabilities, and related synthetic values (risk measures in particular)
- ▷ *RM actions* ⇒ risk classification and product design and pricing

Don't forget: the first phase of the RM process is *objective setting*; if

- raising the market share
- creating clients' value

are among the objectives, then product design and related appropriate pricing are the most critical issues

Basic references

Where links are provided, they were active as of the time this presentation was completed but may have been updated since then.

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