Milliman

Population Dynamics

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Alexandre Boumezoued, PhD – Milliman Paris Office, R&D

What is population dynamics?

- The field of population dynamics modelling aims at understanding **the time behavior of the population structure**
  - **Key notions:**

Structure	**Age pyramid**	**Heterogeneity**	**Dynamics**
A population is a complex system made of individuals with own **ages and characteristics**	The age pyramid keeps track of the number of individuals with a given age at the **deepest age scale**	Population heterogeneity arises as **several characteristics** are represented, such as gender, income, marital status or health state	The population structure evolves dynamically due to **demographic events**, such as aging, death, birth, migration or characteristics changes during life

Why population dynamics?

- The virtue of population dynamics is to give **insights on how individual behaviors impact aggregate quantities**
  - In particular, it allows to understand the **interplay between mortality and fertility** and the **impact of heterogeneity**
- Two levels of analysis of the population structure allow to answer specific questions:
- First part of the presentation: **age pyramid patterns**

  How to improve mortality estimates with fertility data?

- Second part of the presentation: **population heterogeneity**

  How can heterogeneity create (true) cohort effects?

- A. Boumezoued, 2016. **Improving HMD mortality estimates with HFD fertility data.** HAL preprint: https://hal.archives-ouvertes.fr/hal-01270565v1
- H. Bensusan, A. Boumezoued, N. El Karoui, and S. Loisel. 2015. **Bridging the gap from microsimulation practice to population models: a survey.** Preliminary version as Chapter 2 of A. Boumezoued PhD thesis available at https://tel.archives-ouvertes.fr/tel-01307921/document.

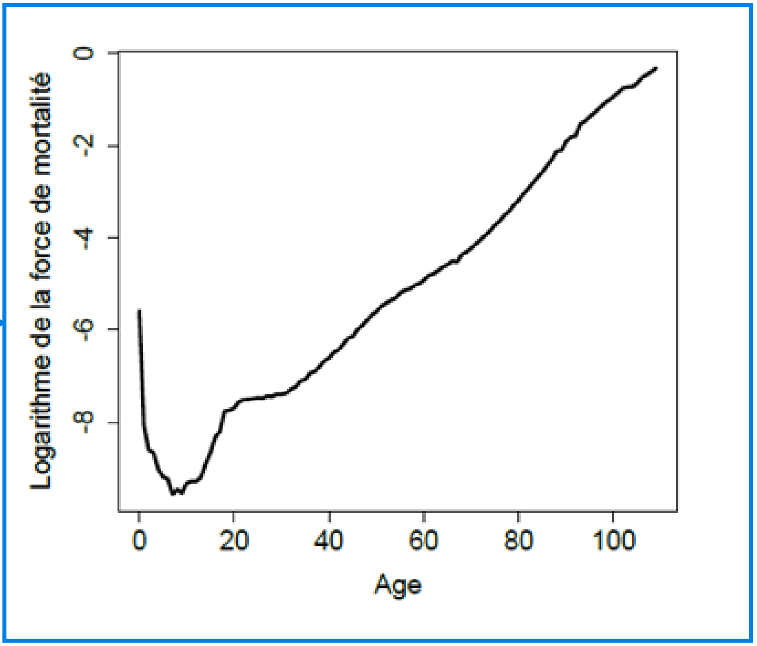
Agenda

1	**Improving mortality estimates with fertility data**
2	How can heterogeneity create (true) cohort effects?

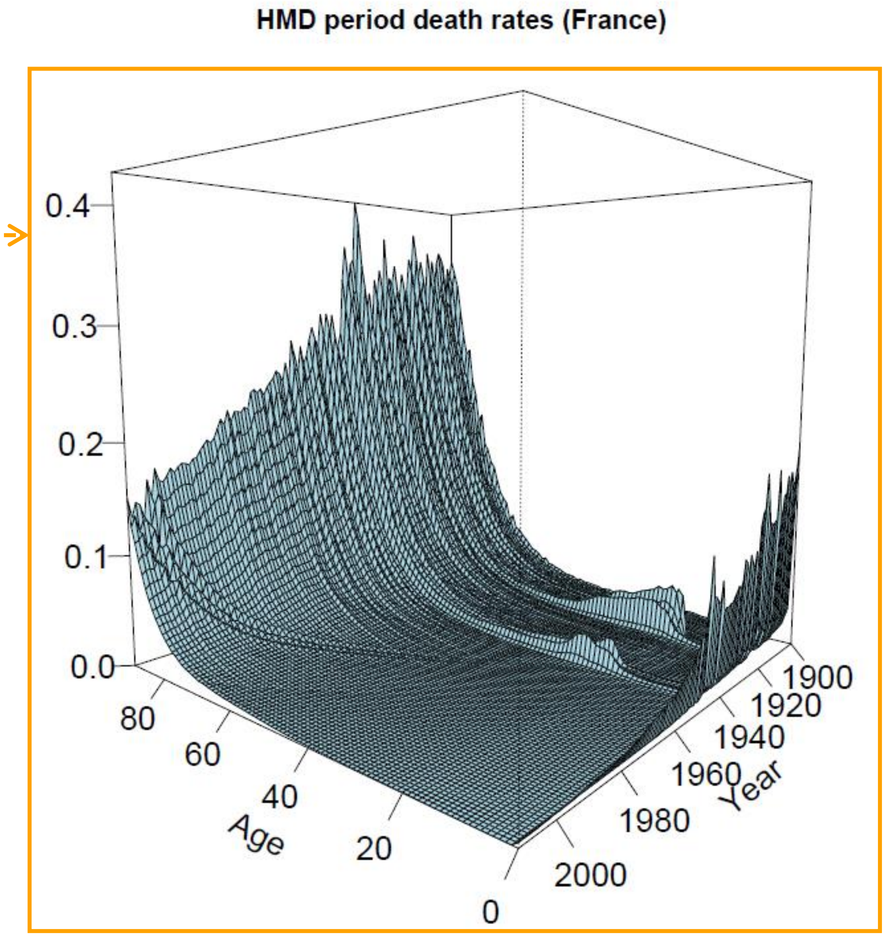
Improving mortality with fertility

Death rate over age and time

- Death rates show particular **age shape** and **time pattern**



Logartihm of death rates for France in 2008 over age



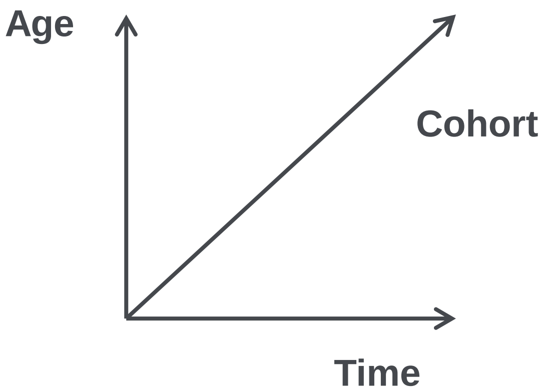
Death rates for France over age and time

Improving mortality with fertility

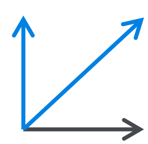
National mortality tables

- How to estimate the **mortality rate** based on national population data?
  - The statistical inference of a death rate with two crossing dimensions (age and time) is an **old (Lexis, 1875) and still challenging estimation problem**
- In practice, individuals are grouped into **age and time blocks**, and the death rate is assumed to be constant on each block
  - This leads to the so-called **Lexis diagram**

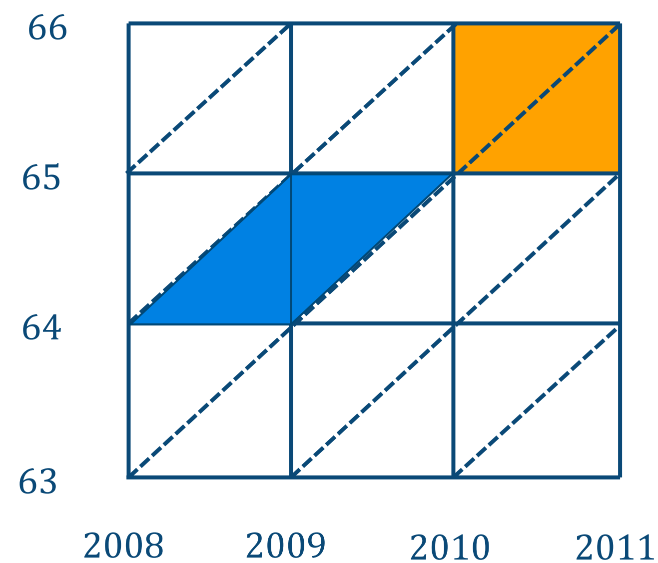
Three directions of analysis

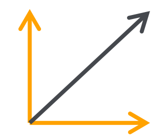


Cohort tables
=
Death rate assumed constant over parallelograms



Period tables
=
Death rate assumed constant over squares

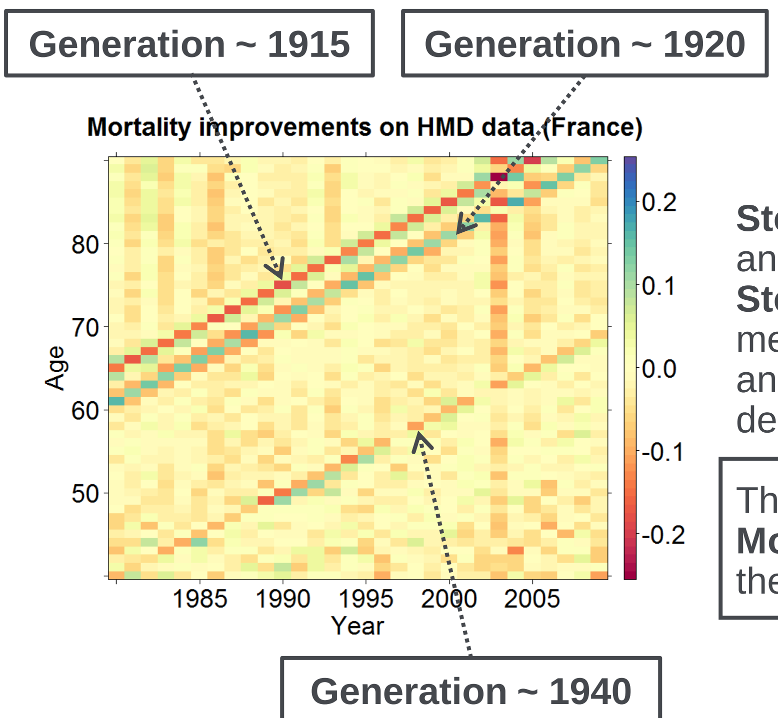




Improving mortality with fertility

Mortality improvement rates and “cohort effects” (ex: France)

- **Period tables** are useful to study the dynamics of mortality over time
  - Period mortality rate for age x and year t denoted $\mu(x,t)$
  - Improvement rates $r(x,t) = \frac{\mu(x,t+1)-\mu(x,t)}{\mu(x,t)}$ are used to observe particular patterns
    - Clear **« cohort effects »** can be observed for specific generations (born around 1915, 1920 and 1940)



Literature on mortality data reliability

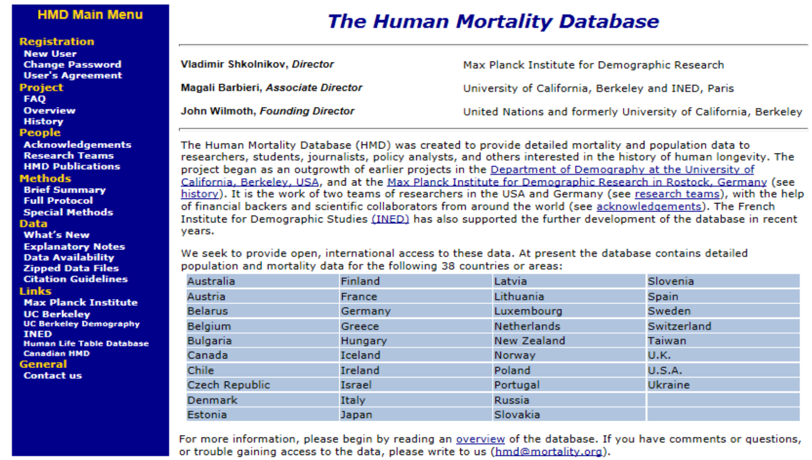
Step 1: Richards (2008) conjectured that the 1919 cohort effect for England and Wales is an **anomaly** in the mortality table due to **erratic birth patterns**
Step 2: Cairns, Blake, Dowd & Kessler (2016) analyzed the ONS methodology and confirmed the conjecture by Richards; they used England and Wales **monthly fertility data** to detect anomalies in the computation of death rates

This talk is based on Boumezoued (2016) and focuses on the **Human Mortality Database**, showing that **these anomalies are universal** and that the **Human Fertility Database** can be processed to correct such errors

Improving mortality with fertility

A reference: the Human Mortality Database*

- Since its launch in 2002, the Human Mortality Database has become the **reference provider** of mortality estimates (both period and cohort tables) given in an homogenous format for several countries



HMD Main Menu

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The Human Mortality Database

Vladimir Shkolnikov, *Director*	Max Planck Institute for Demographic Research
Magali Barbieri, *Associate Director*	University of California, Berkeley and INED, Paris
John Wilmoth, *Founding Director*	United Nations and formerly University of California, Berkeley

The Human Mortality Database (HMD) was created to provide detailed mortality and population data to researchers, students, journalists, policy analysts, and others interested in the history of human longevity. The project began as an outgrowth of earlier projects in the Department of Demography at the University of California, Berkeley, USA, and at the Max Planck Institute for Demographic Research in Rostock, Germany (see history). It is the work of two teams of researchers in the USA and Germany (see research teams), with the help of financial backers and scientific collaborators from around the world (see acknowledgements). The French Institute for Demographic Studies (INED) has also supported the further development of the database in recent years.

We seek to provide open, international access to these data. At present the database contains detailed population and mortality data for the following 38 countries or areas:

Australia	Finland	Latvia	Slovenia
Austria	France	Lithuania	Spain
Belarus	Germany	Luxembourg	Sweden
Belgium	Greece	Netherlands	Switzerland
Bulgaria	Hungary	New Zealand	Taiwan
Canada	Iceland	Norway	U.K.
Chile	Ireland	Poland	U.S.A.
Czech Republic	Israel	Portugal	Ukraine
Denmark	Italy	Russia	
Estonia	Japan	Slovakia	

For more information, please begin by reading an overview of the database. If you have comments or questions, or trouble gaining access to the data, please write to us (hmd@mortality.org).

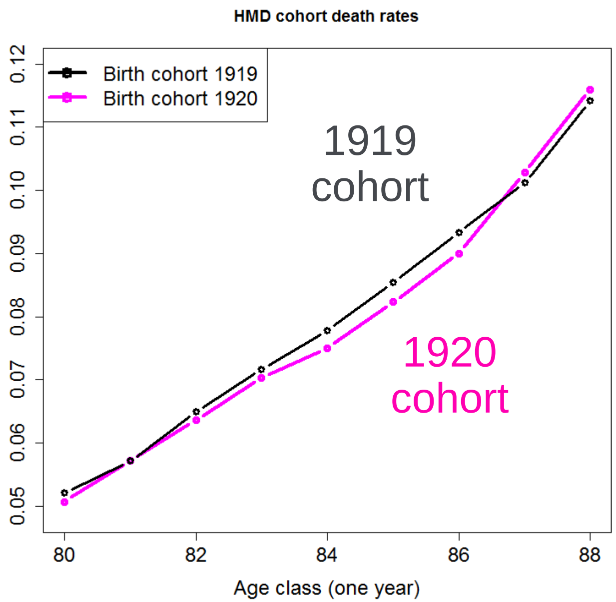
- Possible anomalies in **period mortality tables** are already suggested in the HMD technical note *(Wilmoth et al. 2007):*

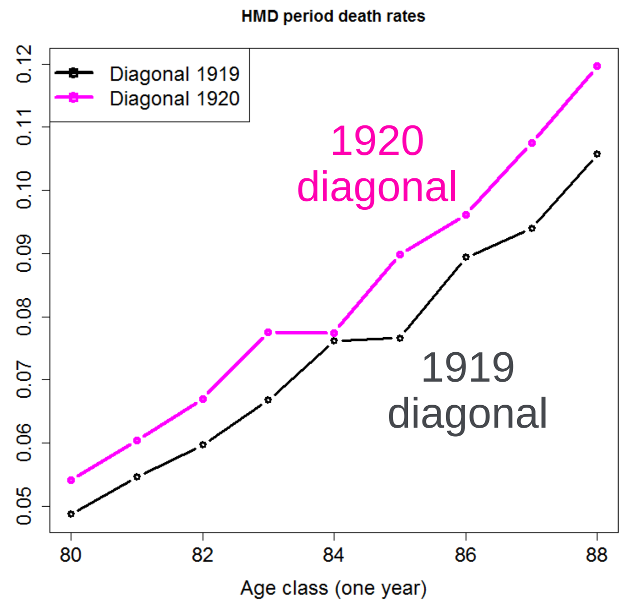
" The assumption [of uniform distribution of births] is violated most severely in situations where there are rapid changes in the size of successive cohorts, owing to **fluctuations in the birth series many years before**. The worst situation is when a sharp discontinuity in births occurs in the middle of one calendar year, creating a cohort that is "heavy" at one end and "light" at the other. **We have not attempted to correct our mortality estimates for the error introduced by such occurrences, which may result in artificially elevated or depressed levels of mortality along a diagonal of the Lexis diagram that follows the cohort(s) in question. The user should be aware of this possibility and not misinterpret the data.**"

**Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org or www.humanmortality.de (data downloaded on October 2015).*

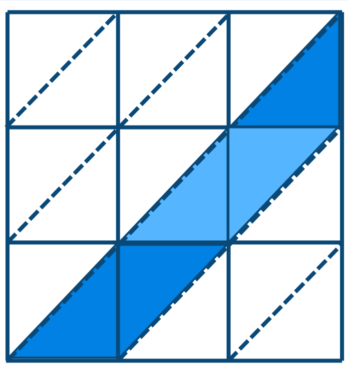
Improving mortality with fertility

Inversion of cohort and period tables: example for France (1/2)

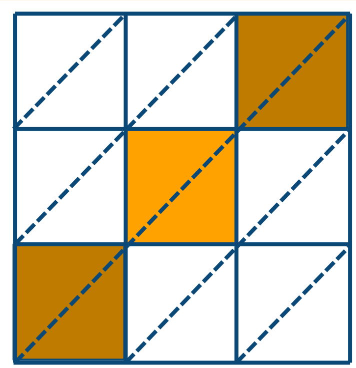




Cohort table



Period table



Improving mortality with fertility

How to properly estimate the period death rate?

- Quantity of individuals with **exact age a at exact time s: $g(a,s)$**
- The **death rate $\mu(a,s)$** drives the evolution of each cohort over time
  - Let $g(0,s)$ be given (the number of newborns at time s)
  - The number of survivors at age a in the cohort born at time s is given as:

$$g(a, s+a) = g(0,s)\exp\left\{-\int_0^a \mu(u, s+u)du\right\}$$

  - Differentiation by age and time leads to the **aging term** of the McKendrick-Von Foerster equation (1926)

$$(\partial_a + \partial_s)g(a,s) = -\mu(a,s)g(a,s)$$

= $d(a,s)$ = Number of deaths at (exact) age a and time s

- **Statistical estimation:** the period death rate is assumed to be constant on squares
  - That is $\mu(a,s) = \mu(x,t)$ for each $a \in [x, x+1)$ and $s \in [t, t+1)$
  - **Under this assumption, one recovers the classical formula of the estimated death rate, as the number of deaths divided by the so-called exposure-to-risk**

$$D(x,t) = \int_t^{t+1}\int_x^{x+1} d(a,s)\,\mathrm{dads} = \mu(x,t)\int_t^{t+1}\int_x^{x+1} g(a,s)\,\mathrm{dads}$$

$$= \mu(x,t)\int_t^{t+1} \underbrace{P(x,s)}\,\mathrm{ds}$$

Population at time s aged x last birthday

Exposure-to-risk = total time lived in year t by individuals aged x last birthday

Improving mortality with fertility

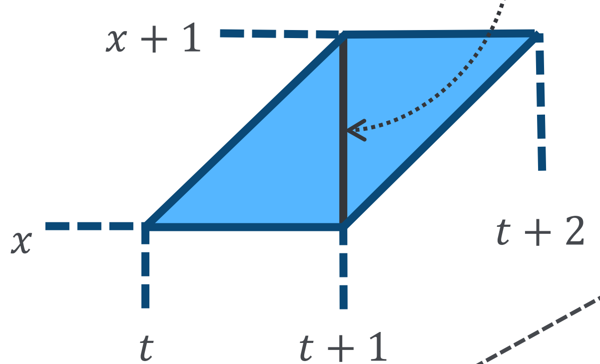
How is the exposure-to-risk estimated in the HMD?

Number of deaths → Exposure-to-risk

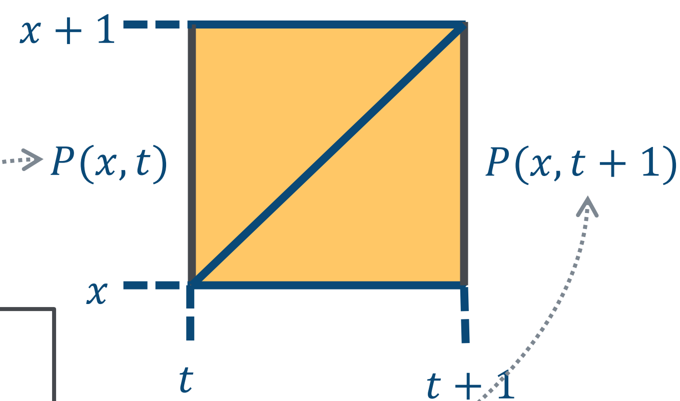
Death rate estimation : $\hat{\mu}(x,t) = D(x,t)/E(x,t)$

COHORT TABLE

HMD Cohort Exposure-to-risk = Population estimate at a given time + Small correction based on number of deaths in each triangle



PERIOD TABLE



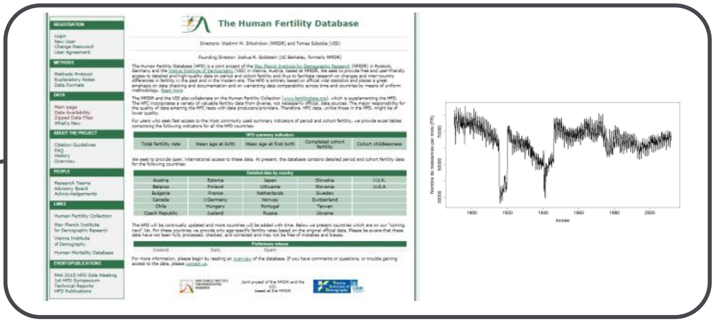
HMD Period Exposure-to-risk = Approximation under the assumption of uniform distribution of births on consecutive years $\frac{1}{2}[P(x,t)+P(x,t+1)]$ + Small correction based on number of deaths in each triangle

⚠ $\approx \int_t^{t+1} P(x,s)\,ds$

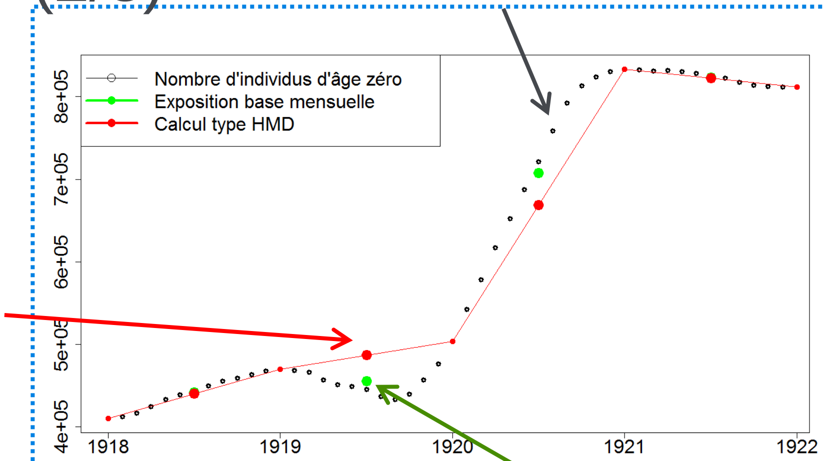
Improving mortality with fertility

Processing the HFD to correct the HMD (2/3)

*Extract monthly fertility records from the **Human Fertility Database***

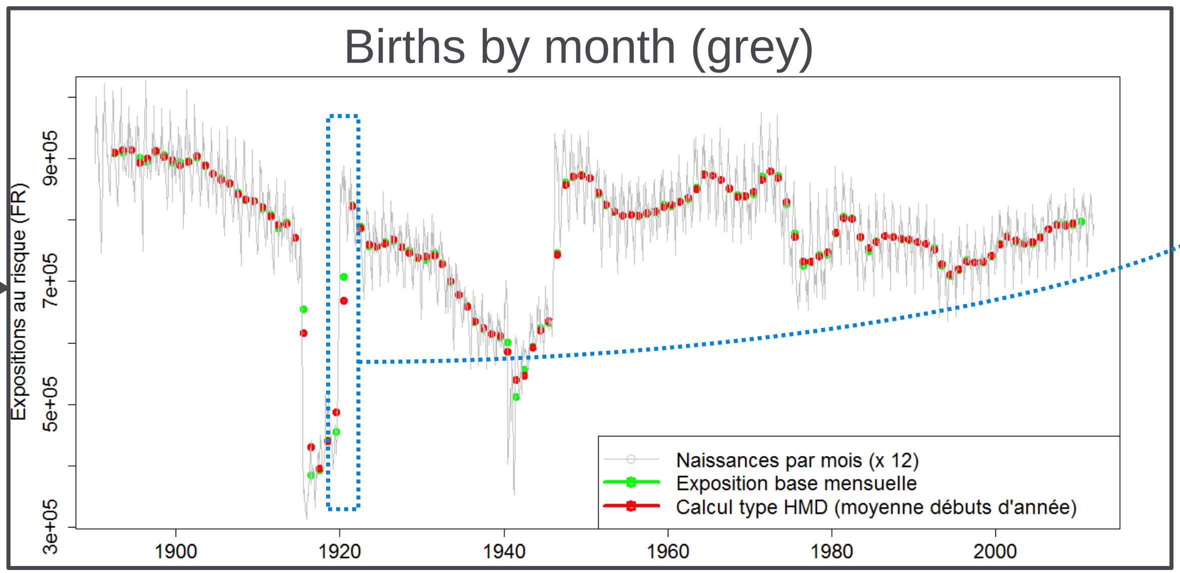


Monthly population size with age zero last birthday (HFD)



HMD-type exposure-to-risk = linear interpolation

Refined exposure-to-risk (based on HFD)

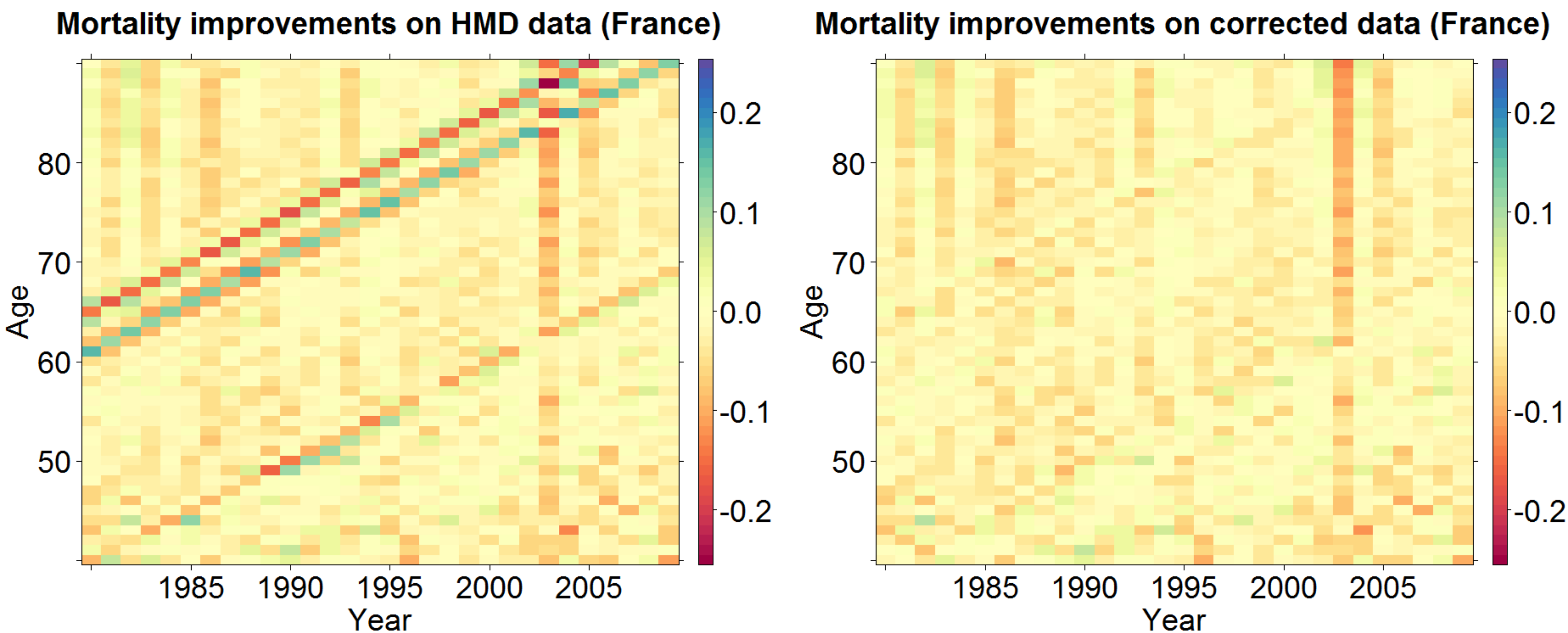


The **bias** in exposure-to-risk computation is high in periods in which births are fluctuating

Explanation: shocks in birth patterns create **convexity** in population numbers => HMD linear approximation is no longer valid

Improving mortality with fertility

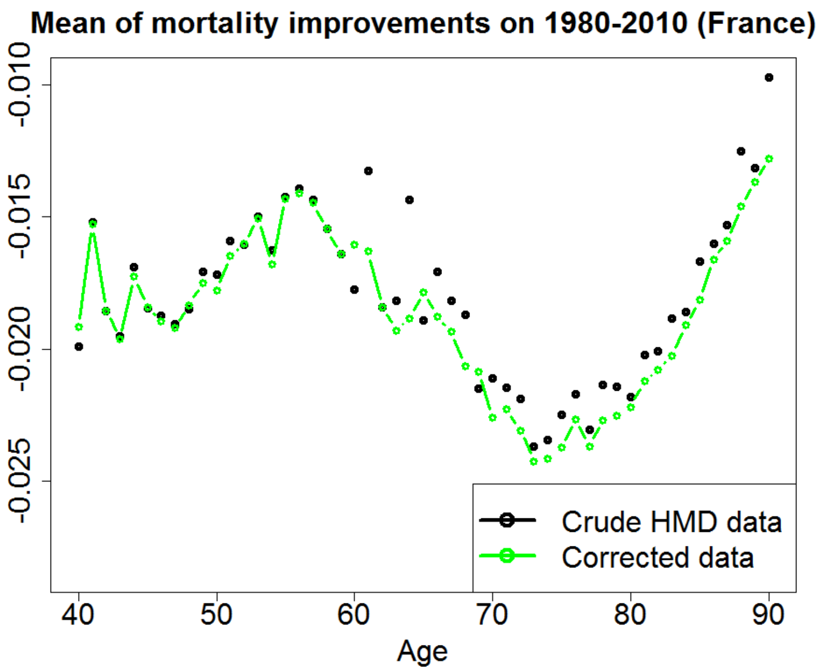
Isolated cohort effects as data anomalies



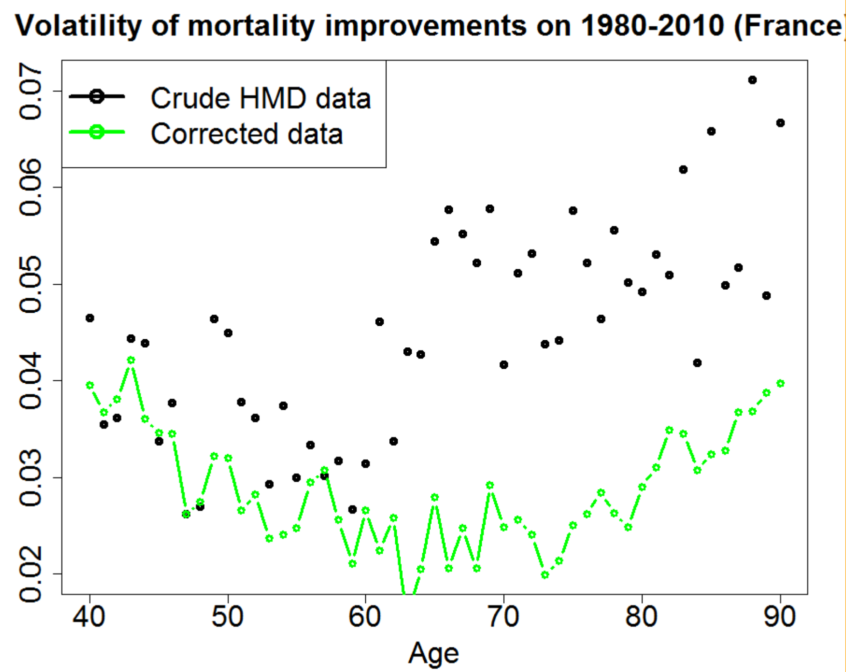
Improving mortality with fertility

Improvement rates properties after data correction

Average mortality improvements by age are smoothed after data correction



Volatility of mortality improvement is reduced at high ages after correction



Agenda

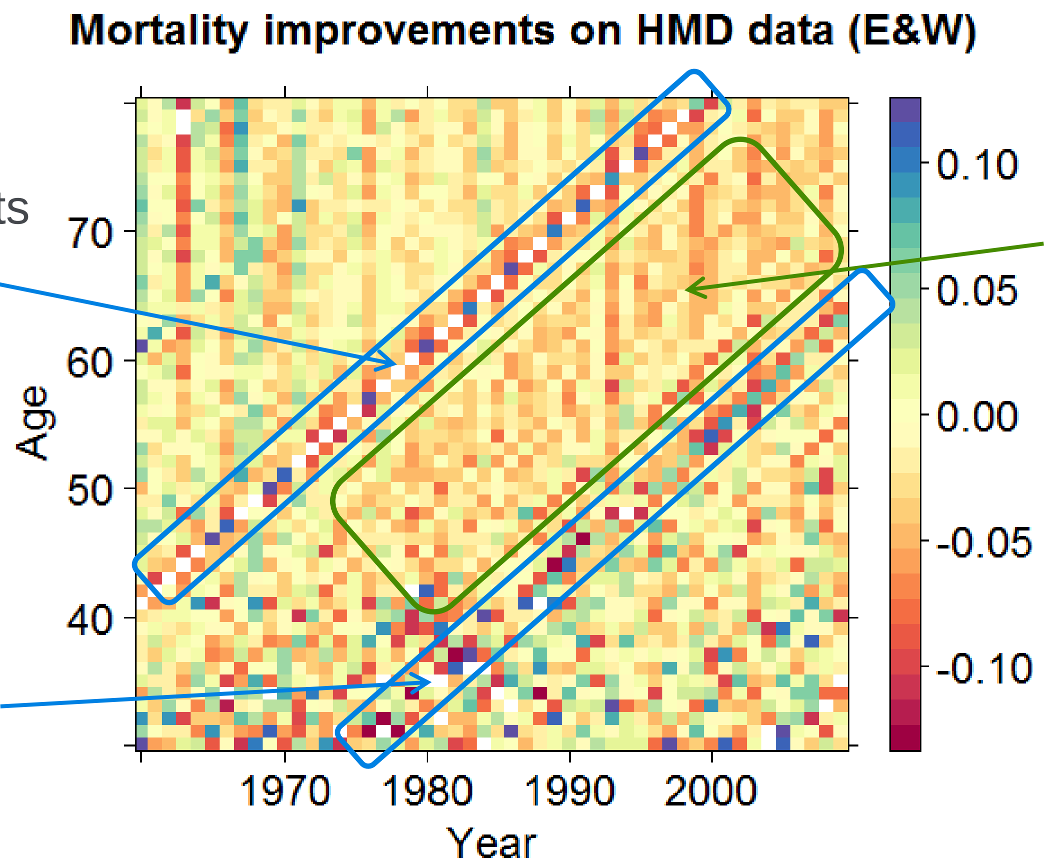
1. Improving mortality estimates with fertility data
2. **How can heterogeneity create (true) cohort effects?**

How can heterogeneity create cohort effects?

The example of England & Wales

- England & Wales is **known to embed cohort effects**

Isolated cohort effect = **data anomalies** of the 1919-1920 cohorts



Extended cohort effect = **known as the Golden Cohort**: the generations born between 1925 and 1945 show particularly high longevity improvements

Isolated cohort effect = **data anomalies** of the cohorts born around 1945

How can heterogeneity create cohort effects?

Demographic insights on the Golden Cohort

- Reference: *The Cohort Effect: Insights And Explanations*, 2004, R. C. Willets
- The Golden Cohort (births between 1925 and 1945) has experienced **higher longevity improvements** compared to earlier and later generations. Possible explanations:
  - Impact of World War II on earlier generations
  - Changes on smoking prevalence: tobacco consumption in later generations
  - Impact of diet in early life
  - Post World War II welfare state

MORTALITY INSIGHTS

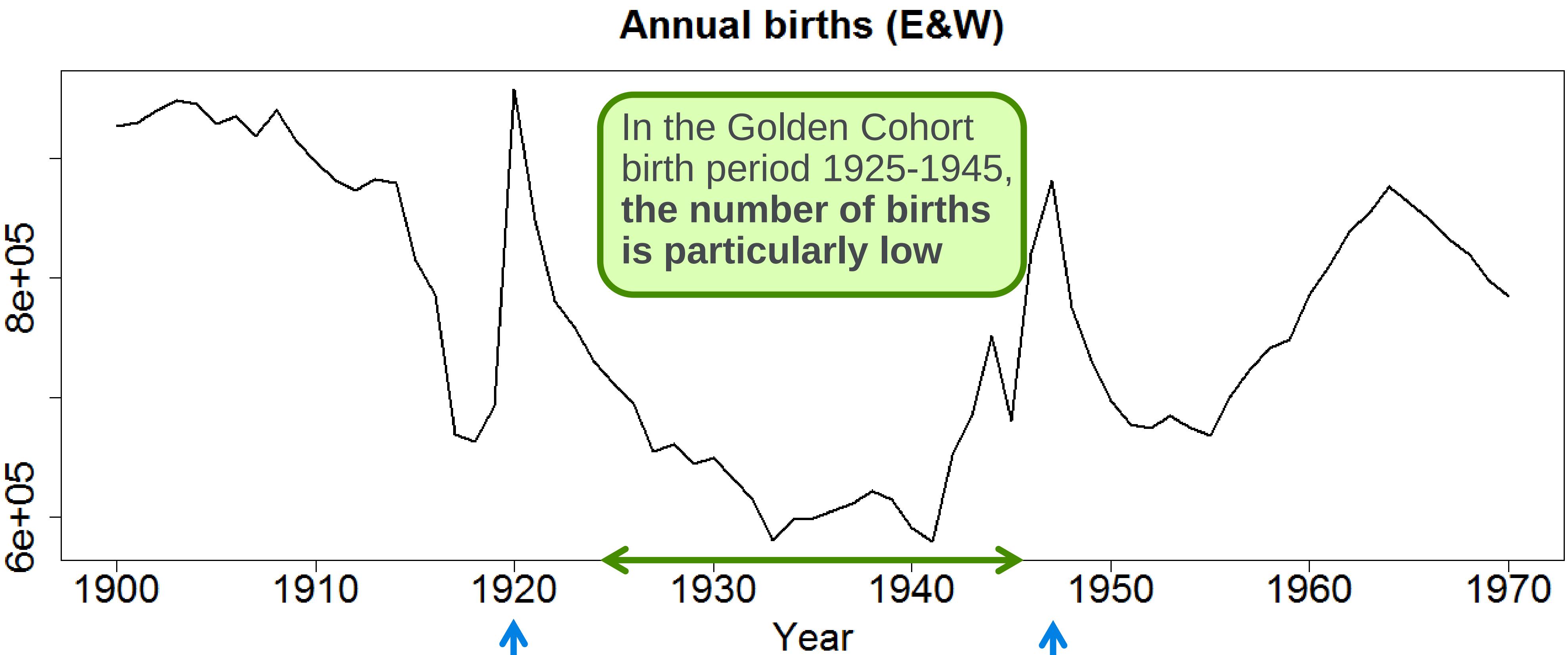
  - Patterns of birth rates

=> Again, **fertility insights** seem to be a promizing direction for a further understanding of **unusual mortality patterns**

FERTILITY INSIGHT

How can heterogeneity create cohort effects?

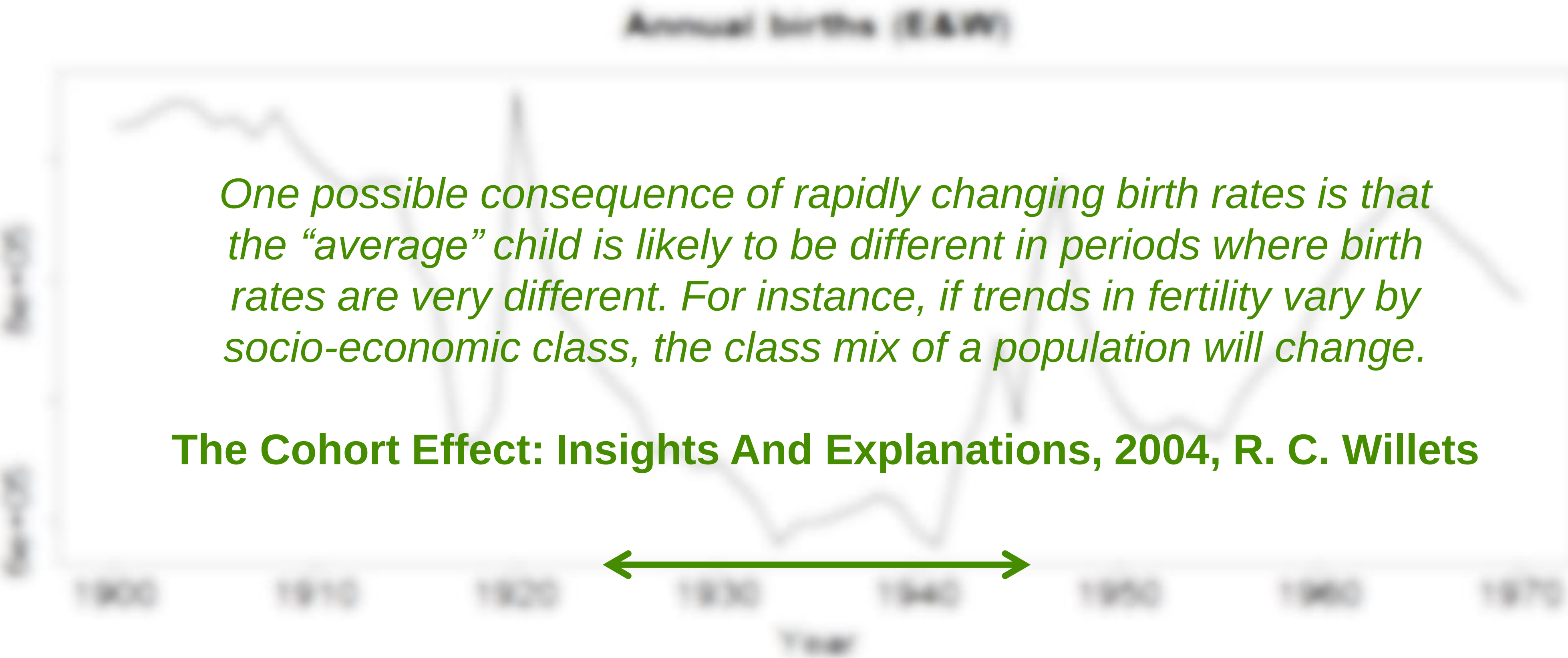
Fertility patterns and cohort effects, again (1/2)



Spikes in birth numbers have already been identified to produce **data anomalies** (Section 1 of the presentation)

How can heterogeneity create cohort effects?

Fertility patterns and cohort effects, again (2/2)



One possible consequence of rapidly changing birth rates is that the “average” child is likely to be different in periods where birth rates are very different. For instance, if trends in fertility vary by socio-economic class, the class mix of a population will change.

The Cohort Effect: Insights And Explanations, 2004, R. C. Willets

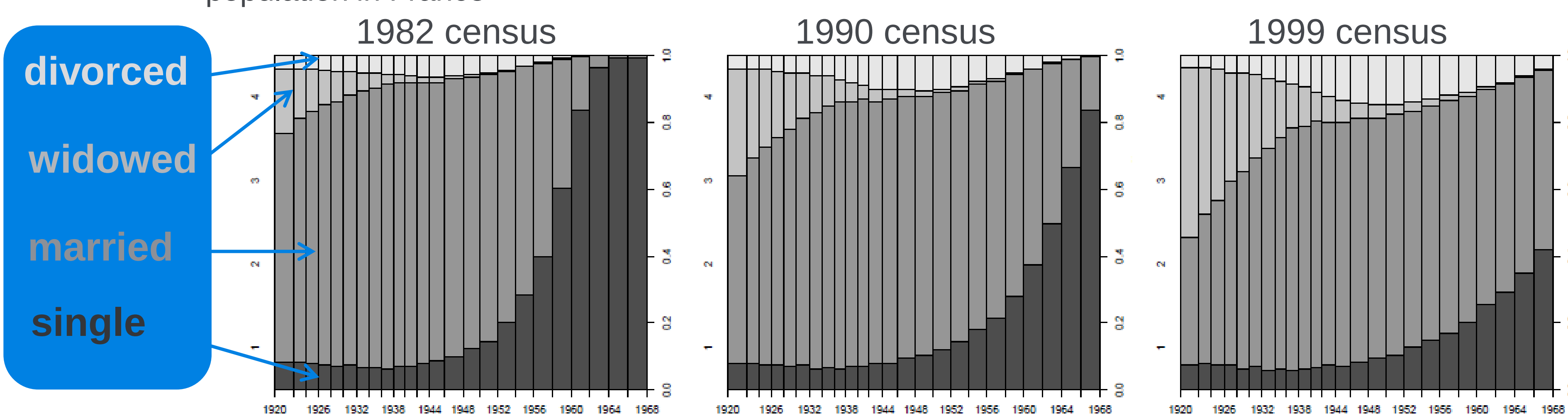
How can heterogeneity create cohort effects?

The need for an heterogeneous population model

- Heterogeneous population models account for the fact that **individuals with same age at the same time may behave differently**
  - This amounts to introduce **specific characteristics** (covariates) x which impact birth and death rates, and to decompose the total population with age a at time t into sub-groups:

$$g(a,t) = \sum_x g(x,a,t)$$

  - Heterogeneity introduces a **strong complexity** in the demographic process
    - Example (from Boumezoued, El Karoui & Loisel, 2016): time evolution of the marital status of the female population in France



Years of birth from 1920 to 1968

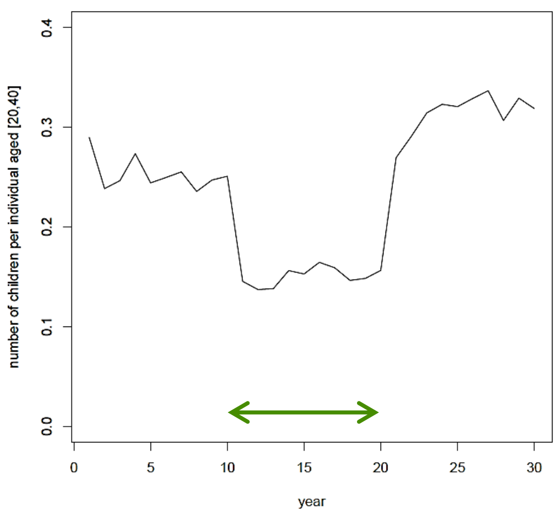
How can heterogeneity create cohort effects?

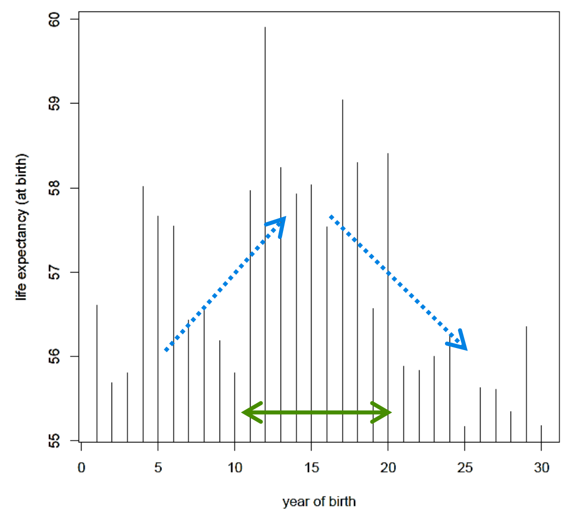
Insights from simulation of the population dynamics model

- **Aim:** run Willets' insights based on a birth-death population model
- Consider an **heterogeneous population model**: how to make **aggregate mortality vary** while **individual mortality remains** the same?
  - **Simulation of the population dynamics model** with two groups (and perfect heredity):
    - **Group 1:** low mortality and fertility individuals
    - **Group 2:** high mortality and fertility individuals but…

…**during a period 10-20 (arbitrary time scale), individual fertility is reduced for Group 2 (this is the only individual time change in the model)**

Aggregate birth rate over time





« Cohort effect » for life expectancy by year of birth computed on the whole population

Conclusion

So, why population dynamics?

- To get insights on **aggregate mortality patterns**, it is needed to look at the population at a refined scale
- Population dynamics models allow to quantify the evolution over time of the **population age pyramid and heterogeneous structure**
- The underlying population modelling framework is a **flexible toolbox** leading to **new insights** on mortality patterns

1. Birth patterns at a refined time scale → **correct abnormal cohort effects**
2. Heterogeneous birth patterns → **run insights on the Golden Cohort effect**

References

- A. Boumezoued, 2016. **Improving HMD mortality estimates with HFD fertility data.** HAL preprint: https://hal.archives-ouvertes.fr/hal-01270565v1
- H. Bensusan, A. Boumezoued, N. El Karoui, and S. Loisel. 2015. **Bridging the gap from microsimulation practice to population models: a survey.** Preliminary version as Chapter 2 of A. Boumezoued PhD thesis available at https://tel.archives-ouvertes.fr/tel-01307921/document.
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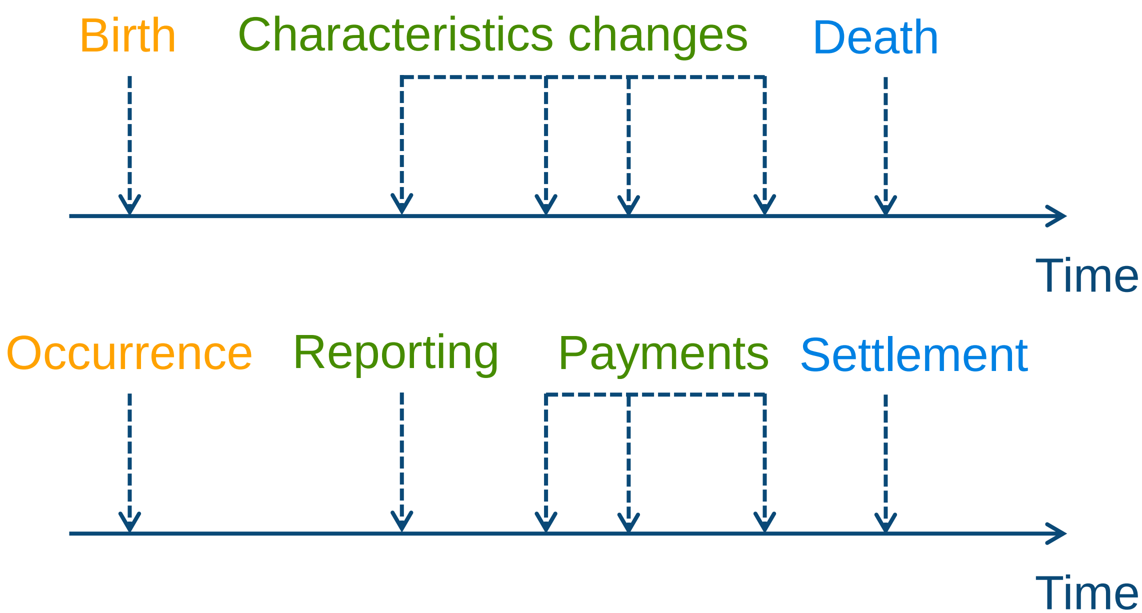
Thank you

alexandre.boumezoued@milliman.com

Bonus: A quick journey into individual reserving

- Aim of this slide is to illustrate a possible use of population dynamics modelling beyond life insurance topics
  - As an example, let us illustrate the analogy with the **claims development process in non-life insurance** (already suggested by various authors in this field)



The set of claims behaves as a population process