ON MANAGING CENTRAL BRANCH RISK NETWORKS, USING SCALE FUNCTIONS

Florin Avram

Abstract

The study of the interaction between a reinsurer and several insurers, or between a central branch (CB) and several subsidiary companies suggest the concept of CB risk networks, and many interesting problems.

Keywords: risk networks, valuation problem, dividends optimization, liquidation penalty, spectrally negative processes, scale functions

1 Central branch risk networks

Definition 1. A central branch (CB) risk network *is formed from:*

1. *Several spectrally negative subsidiaries* $X_i(t), i = 1, \ldots, I$, *which must be kept above certain prescribed levels* o_i *by bail-outs from a central branch (CB)* $X_0(t)$, *or be liquidated when they go below* o_i.

2. *The reserve of the CB is a spectrally negative process denoted by* $X_0(t)$ *in the absence of subsidiaries, and by* $X(t)$ *after subtracting the bailouts. The ruin time*

$$\tau = \tau_0^- = \inf\{t \geq 0 : X(t) < 0\}$$

causes the ruin of the whole network and leads to a severe penalty.

3. *The CB must also cover a certain proportion* $\overline{\alpha}_i = 1 - \alpha_i$ *of each claim* $C_{i,j}$ *of subsidiary* i, *leaving the subsidiary to pay only* $\alpha_i C_{i,j}$, *where* $\alpha_i \in [0, 1]$ *are called proportional reinsurance retention levels.*

Remark 1. *For a CB network, the boundaries* $u_i = o_i, i = 1, ..., I$ *are reflecting and* $u_0 = 0$ *is absorbing.*

Two important characteristics of a CB network are:

1. the ruin probability transform:

$$S_q(\boldsymbol{u}, \boldsymbol{\theta}) := E_{\boldsymbol{u}} \left[e^{-q\tau + <\boldsymbol{\theta}, \boldsymbol{X}(\tau)>}; \tau < \infty \right],$$

$$\boldsymbol{X}(t) = (X_0(t), X_1(t), ..., X_I(t)). \quad (1)$$

2. the optimal discounted dividends until ruin:

$$V^F(\boldsymbol{u}) := \sup_{\pi = (R_0, R_1, \ldots, R_I)} E_{\boldsymbol{u}} \int_0^\tau e^{-qt} \left(\sum_{i=0}^I dR_i(t) \right),$$

where R_i denotes the nonnegative cumulative dividends/consum process paid by the i-th branch.

More generally, τ could be replaced by other stopping times, like the drawdown time

$$\tau_\xi := \inf\{t \geq 0 : X_t \leq \xi \sup_{0 \leq s \leq t} X_s\},$$

where $\xi \in (0, 1)$ is a fixed constant.

1.1 Heuristic valuation using equilibrium line policies

A natural approach for **evaluating financial companies**, going back to de Finetti [DF57] and Modigliani and Miller [MM61] is to consider the optimal expected discounted cumulative dividends/optimal consumption until ruin (2) – see [LST14] for further references on this venerable approach.

If the liquidation time τ is also optimized

$$\mathcal{I}^F(\boldsymbol{u}) := \sup_{\pi=(R_0,R_1,\ldots,R_I,\tau)} E_{\boldsymbol{u}} \int_0^{\tau} e^{-qt} \left(\sum_{i=0}^{I} dR_i(t) \right), \quad (3)$$

the result $\mathcal{I}^F(\boldsymbol{u})$ is a Gittins type valuation index. We will propose now a heuristic multi-dimensional valuation index inspired by the remarkable fact that central branch network problems admit occasionally explicit answers, if the retention levels are small enough [APP08a, APP08b, BCR11, AMP16].

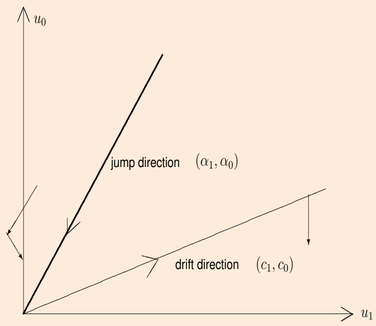

Figure 1: Geometrical considerations

We recall from the papers above that when $I = 1$ and

$$c_0 \leq c_1 \frac{\overline{\alpha}_1}{\alpha_1},$$

i.e. if the angle of the vector $\boldsymbol{\alpha} = (\alpha_1, \overline{\alpha}_1)$ with the u_1 axis is bigger than that of $\boldsymbol{c} = (c_1, c_0)$, then the lower cone

$$\mathcal{C} := \{0 \leq u_0 \leq u_1 \frac{\overline{\alpha}_1}{\alpha_1}\}$$

contains $\boldsymbol{c}$ and is invariant with respect to the stochastic flow, and the ruin probability is a classic one-dimensional ultimate ruin probability

$$\Psi(u_1, u_0) = \Psi(\alpha_1 \frac{u_0}{\overline{\alpha}_1}, u_0) := \Psi_0(u_0), \forall u_0$$

Turning now to several dimensions several dimensions, it is easy to check that:

Lemma 1. *The cone*

$$\mathcal{C} := \{0 \leq u_0 \leq u_i \frac{\overline{\alpha}_i}{\alpha_i},\ \overline{\alpha}_i = 1 - \alpha_i,\ i = 1, ..., I\}$$

is invariant under the "(extra) cheap reinsurance" condition

$$c_0 \leq c_i \frac{\overline{\alpha}_i}{\alpha_i}, i = 1, ..., I. \tag{4}$$

The boundary edge

$$u_1 \frac{1 - \alpha_1}{\alpha_1} = ... = u_i \frac{1 - \alpha_i}{\alpha_i} = u_0,\ i = 1, ..., I, \tag{5}$$

to be called "**equilibrium line**", plays a prominent role in two recent papers, [BB11] § and [AMP16], who solved the optimal dividends problem in the (extra) cheap reinsurance two-dimensional case $c_1 \frac{1-\alpha_1}{\alpha_1} > c_0$. The last paper showed that:

---

§ who computed an explicit value function maximizing an expected exponential utility at a fixed terminal time for multi-dimensional reinsurance model under the "cheap reinsurance" assumption that the drifts point along the line $c_1 \frac{1-\alpha_1}{\alpha_1} = ... = c_I \frac{1-\alpha_I}{\alpha_I}$.

1. Starting from the equilibrium line, the optimal policy is to stay on this line by cashing the excess income of the subsidiary as dividends.

2. Starting from points away from the equilibrium line, in the cheap reinsurance case, the optimal policy is to reach the equilibrium line by one lump sum payment.

3. In the extra cheap reinsurance case, the optimal policy is more complicated, when starting in a certain egg-shaped subset of the non-invariant cone (where parts of the premia are cashed, following a ”shortest path”, in some sense).

The first two findings prompt us to introduce multi-dimensional ”equilibrium line” policies for (extra) cheap reinsurance networks, under which the network follows this line in the absence of claims, by **subsidiaries cashing part of their premia as dividends**. Subsequently, whenever the CB or one subsidiary drop below, **all the other subsidiaries** reduce their reserves by **lump sum dividend** taking, bringing back the process on the equilibrium line.

Remark 2. *These strategies may not be optimal; however, since they stipulate that the subsidiaries just ”***accompany deterministically***” the CB, the value of the network expected dividends decomposes as a sum of one-dimensional quantities– see next Lemma.*

Lemma 2. *For a general CB network, and a fixed admissible dividends process $R_0(t)$, the de Finetti value function for the equilibrium policy associated to $\pi = (R_0, \tau)$ is:*

$$V_\pi^F(x) = E_x\Bigg[\int_0^\tau e^{-qt}\Big[dR_0(t) + \widetilde{c}dt - \gamma dX_0(t) - \sum_{i=1}^{I}(\gamma\frac{\overline{\alpha}_i}{\alpha_i} - 1)d$$

where

$$\gamma = \sum_{i=1}^{I}\frac{\alpha_i}{\overline{\alpha}_i},\ \widetilde{c} = \gamma\sum_{i=1}^{I}c_i\frac{\overline{\alpha}_i}{\alpha_i}.$$

Optimizing dividends reduces thus to a one-dimensional problem.

2 Valuation of spectrally negative Lévy subsidiaries using dynamic indices built from the scale functions

Consider a subsidiary with liquidation value $w(x)$, and the value function of a policy $\pi = (R, R_*, \tau)$ involving some dividend process R, bailout process R_*, and stopping time τ. Modify now this value function by subtracting a constant subsidy $\mathcal{I} = \mathcal{I}(x)$ for stopping, and choose this subsidy so that the decision of whether to continue or stop yield equal payoffs. Suppose that $dR_*(t)$ consists of a single payoff at the liquidation/reevaluation time τ.

Then, the valuation index is provided by the equation:

$$\sup_{\pi} E_x \Big[\int_0^{\tau} e^{-qt} \Big(dR(t) + dR_*(t) \Big) \Big] - \mathcal{I} = w(x)$$

$$\Longrightarrow \mathcal{I}(x) = \sup_{\tau} E_x \left[\int_0^{\tau} e^{-qt} dR(t) + e^{-q\tau} w(X(\tau)) \right] - w(x)$$

As a simplification of this optimal stopping problem, suppose that the stopping time is prescribed by forced stopping, for example at $\tau = \tau_o^-$.The result is a modified De-Finetti objective

$$V^F(x) = E_x \left[\int_0^{\tau} e^{-qt} dR(t) + e^{-q\tau} w(X(\tau)) \right] - w(x) \quad (6)$$

With linear liquidation costs $w(x) = \begin{cases} kx - K, & x < 0 \\ x - K, & x \geq 0 \end{cases}$, where $K \geq 0$ is a penalty for quick liquidation, and $\tau = \tau_0^-$, this problem was essentially studied in [LR10, APP15]. The approximate index is then:

$$V^F(x) = \mathcal{S}_w(x) + W_q(x) \frac{1 - \mathcal{S}_w'(b)}{W_q'(b)} - w(x), \tag{7}$$

$$\mathcal{S}_w(x) = kZ_{1,q}(x) - KZ_q(x),\ Z_{1,q}(x) = \overline{Z}_q(x) - p\overline{W}_q(x)$$

The optimal b is typically the last maximum of the barrier function

$$G(b) = \frac{1 - \mathcal{S}_w'(b)}{W_q'(b)}. \tag{8}$$

The optimization of the bailout point o has been less studied, and deserves further attention.

Remark 3. *Several variations of this index may be obtained replacing absorption at τ by Parisian absorbtion or reflection, and by adding refraction or other boundary mechanisms; the valuation index $\mathcal{I}$ is again given by* (7)*, once one uses the appropriate scale functions $W, Z, \mathcal{S}_w$ [KL10, AIZ14, APP15, APY16].*

3 Eight first passage laws for Poissonian (Parisian) detection of insolvency

A useful type of models developed recently [AIZ14, AI15, APY16] assume that insolvency is only **observed periodically**, at an increasing sequence of *Poisson observation times* $\mathcal{T}_r = \{t_i, i = 1, 2, ...,$ the arrival times of an independent Poisson process of rate r, with $r > 0$ fixed.

The analog concepts for first passage times are the stopping times

$$T_b^+ = \inf\{t_i : \ X(t_i) > b\}, \quad T_a^- = \inf\{t_i > 0 : \ X(t_i) < a\} \quad ($$

A **spectrally negative Lévy processes with Parisian reflection** below 0 may be defined by pushing the process up to 0 each time it is below 0 at an observation time T_i.

Remark 4. *Parisian detection below* 0 *is related to the "time spent in the red"*

$$T_{<0} := \int_0^\infty I_{\{X(t)<0\}} dt.$$

$$P_x[T_0^- = \infty] = P_x[T_{<0} < \mathcal{E}(r)] = E_x\left[e^{-rT_{<0}}\right] = p\frac{\Phi_r}{r} Z(x, \Phi_r$$

Proposition 1. *Let X be a spectrally negative Lévy process with Parisian detection below 0 at rate r. Then, for $x \in [0, b]$:*

1. *The* **capital injections/bailouts law for a reflected process, until τ_b^+ [APY16, Cor 3.1 ii)]**, *[IP12, Thm 2]. Let $X^{[0}(t)$ denote the SNMAP process reflected at 0, let $R_*(t) = -(0 \wedge \underline{X}(t))$ denote its regulator at 0, so that $X^{[0}(t) = X(t) + R_*(t)$, and let $\mathbb{E}_x^{[0}$ denote expectation for the process reflected at 0. Then:*

$$L_{*,\theta}(x,b) := \mathbb{E}_x^{[0}[e^{-q\tau_b^+ - \theta R_*(\tau_b^+)}] =$$
$$\begin{cases} Z_{q,r}(x,\theta) Z_{q,r}(b,\theta)^{-1} & \theta < \\ \mathbb{P}^{[0}[\tau_b^+ < T_0] = Z_{q,r}(x,\Phi_{q+r}) Z_{q,r}(b,\Phi_{q+r})^{-1} & \\ := W_{q,r}(x) W_{q,r}(b)^{-1} & \theta = \end{cases}$$

where

$$Z_{q,r}(x,\theta) = \frac{r}{q+r-\kappa(\theta)} Z_q(x,\theta) + \frac{q-\kappa(\theta)}{q+r-\kappa(\theta)} Z_q(x,\Phi_{q+r}),$$

with $\theta = \Phi_{q+r}$ interpreted in the limiting sense.

When $r \to \infty$, $Z_{q,\infty}(x,\theta) = Z_q(x,\theta)$, $W_{q,\infty}(x) = W_q(x)$ and (10) *reduces to classic results [IP12].*

2. *The* **expected discounted dividends until T_0 [AIZ14, (27)]** *are :*

$$V_{q,r}(x,b) = \mathbb{E}_x^{[0,b]}\left[\int_0^{T_0} e^{-qt} dR(t)\right] = W_{q,r}(x) W'_{q,r}(b)^{-1}, \tag{10}$$

where $\mathbb{E}^{[0,b]}$ denotes the law of a process reflected from above at b with Parisian absorbtion at 0, and $R(t)$ denotes the upper regulation at b.

3. *The* **expected discounted dividends with reflection at 0 at Parisian times, until the total bail-outs surpass an exponential variable $\mathcal{E}_\xi$** *satisfy*

$$V^S_{*,\xi}(x,b) = \mathbb{E}_x^{[0,b]}\left[\int_0^{\infty} e^{-qs} 1_{[R_*(s)<\mathcal{E}_\xi]} dR(s)\right]$$
$$= Z_{q,r}(x,\xi) Z'_{q,r}(b,\xi)^{-1}$$

see [AI14, (15)].

4. *The* **severity of Parisian ruin with absorbtion at τ_b^+ [AIZ14, (15)]** *[IP12, Cor 3] is:*

$$S^b(x,\theta) = \mathbb{E}_x\left[e^{\theta X(T_0)}; 1_{T_0<\tau_b\wedge\mathcal{E}_q}\right] =$$
$$Z_{q,r}(x,\theta) - W_{q,r}(x) W_{q,r}(b)^{-1} Z_{q,r}(b,\theta).$$

5. *The* **expected total discounted bailouts at Parisian times up to** τ_b^+ *are given for* $0 \le x \le b$ *by [APY16, Cor 3.2 ii)]:*

$$V_*(x,b) := \mathbb{E}_x^{[0}\left[\int_0^{\tau_b^+} e^{-qt}\mathrm{d}R_*(t)\right]$$
$$= Z_{q,r}(x)Z_{q,r}(b)^{-1}\mathcal{S}(b) - \mathcal{S}(x).$$

where

$$\mathcal{S}(x) = \mathcal{S}_{q,r}(x) = \frac{r}{q+r}\Big(\overline{Z}_q(x) + \frac{\kappa'(0_+)}{q}\Big). \qquad (11)$$

6. *The* **total discounted bailouts at Parisian times over an infinite horizon**, *with reflection at* b *are [APY16, Cor 3.4]:*

$$V_*^S(x,b) = \mathbb{E}_x^{[0,b]}\left[\int_0^{\infty} e^{-qt}\mathrm{d}R_*(t)\right]$$
$$Z_{q,r}(x)Z'_{q,r}(b)^{-1}\mathcal{S}'(b) - \mathcal{S}(x).$$

7. *The* q**-resolvent of doubly absorbed Lévy processes** *may be expressed, in terms of the scale function [BPPR, Thm 2]. Namely, for any Borel set* $B \in [a,b]$*,*

$$\mathbb{E}_x\Big(\int_0^{\tau_a^-\wedge\tau_b^+} e^{-qt}1_{\{X_t\in B\}}\mathrm{d}t\Big)$$
$$= \int_a^b 1_{\{y\in B\}}\Big[\frac{W_{q,r}(x-a)W_{q,r}(b-y)}{W_{q,r}(b-a)} - W_{q,r}(x-y)\Big]\mathrm{d}y$$

8. *The* **dividends- penalty law for a process reflected at b, with Parisian ruin [AIZ14, (23)]**, *[IP12, Thm 6] is:*

$$S_\vartheta^b(x,\theta) := \mathbb{E}_x^{[0,b]}\left[e^{-\vartheta R(T_0)+\theta X(T_0)}; T_0 < \mathcal{E}_q\right] = Z_{q,r}(x,\theta$$
$$W_{q,r}(x)\left(W'_{q,r}(b) + \vartheta W_{q,r}(b)\right)^{-1}$$
$$\left(Z'_{q,r}(b,\theta) + \vartheta Z_{q,r}(b,\theta)\right).$$

When $x = b$, we may factor the transform (**??**)*:*

$$\mathbb{E}_b^{[0,b]}\left[e^{\theta X(T_0)-\vartheta R(T_0)}; T_0 < \mathcal{E}_q\right] = \Omega(\Omega+\vartheta)^{-1}$$
$$\left(Z_q(b,\theta) - \Omega^{-1}\left(\theta Z_q(b,\theta) + (q-\kappa(\theta))\, W_q(b)\right)\right)\frac{r}{r+q-}$$

where

$$\Omega = V_{q,r}^F(b,b)^{-1} = W'_{q,r}(b)W_{q,r}(b)^{-1}$$

By (12)*, $R(T_0)$ and $X(T_0)$ are independent when starting from b, and the former has an exponential distribution with parameter Ω [AIZ14, (23)].*

4 Acceptance-rejection of Lévy subsidiary companies observed at Poissonian times, based on readiness to pay dividends

Even in the one- dimensional case, the final choice of an acceptance-rejection principle is not at all obvious. A first intuition is that an acceptable subsidiary must satisfy the classic positive profit condition

$$p := E_0[X(1)] > 0 \tag{12}$$

or its extension with linear bailout costs [LR10].

Note that the profitability/viability condition of [LR10] is equivalent to

$$G(0) \geq 0,$$

where G is the barrier influence function, and interesting variations may be obtained by replacing absorbtion at 0 with reflection or Parisian reflection, which change the scale functions. The simplicity of all the resulting formulas comes from the fact that the scale functions are only evaluated at 0. This suggested an acceptance-rejection criteria introduced in [AM15, AM16], based on the readiness of subsidiaries to pay dividends at $b = 0$.

Definition 2. *A subsidiary will be called* **efficient** *if the barrier* $b = 0$ *is locally optimal for paying dividends over some interval* $b \in [0, \epsilon), \epsilon > 0$, *i.e. if it holds that*

$$G'(0) \leq 0.$$

The motivation of this condition is that companies satisfying it are functional even in the absence of cash reserves, and can contribute cash-flows to the central branch without having to wait first until their reserves build out; efficiency is thus translated as **readiness to pay dividends**. This criterion turns out to be a useful complement of the **viability** concept $G(0) \geq 0$ (which at its turn generalizes the classic $p \geq 0$).

The next result provides a nontrivial efficiency criteria under the SLG infinite horizon cumulative dividends-bailouts objective with Parisian reflection.

Theorem 1. *a) The SLG value function with Parisian reflection and linear bailout costs kx is:*

$$V_{SLG}(x) = Z_{q,r}(x) Z'_{q,r}(b)^{-1} - k \left(Z_{q,r}(x) Z'_{q,r}(b)^{-1} \mathcal{S}'(b) \right.$$
$$= k\mathcal{S}(x) + Z_{q,r}(x) \frac{1 - k\mathcal{S}'(b)}{Z'_{q,r}(b)}$$

b) The barrier $b = 0$ is a local maximum iff the influence function $G(b) := \frac{1-k\mathcal{S}'(b)}{Z'_{q,r}(b)}$ satisfies

$$G'(0) \le 0 \Leftrightarrow k\Big(\mathcal{S}'(0) Z''_{q,r}(0) - \mathcal{S}''(0) Z'_{q,r}(0)\Big) \le Z''_{q,r}(0)$$
$$\Leftrightarrow k \le (1 + \frac{q}{r}) \frac{\Phi_{q+r} - r W_q(0_+)}{\Phi_{q+r} - (r+q) W_q(0_+)}. \tag{13}$$

In the finite variation case [§] (13) *holds iff*

$$k \le k(q,r) := (1 + \frac{q}{r}) \frac{\Phi_{q+r} - r/c}{\Phi_{q+r} - (r+q)/c}$$

Remark 5. *It may be checked that $k(q,r)$ increases in q from $k(0,r) = 1$ to infinity and thus an inefficient subsidiary with high transaction cost $k > k(q,r)$ may be turned into efficient by increasing the killing q sufficiently.*

[§] in the infinite variation case, the first equation still holds, but the efficiency index does not reflect the distribution, since Φ_{q+r} cancels

References

[AA10] Sören Asmussen and Hansjörg Albrecher. *Ruin probabilities*, volume 14. World Scientific, 2010.

[AI13] Hansjörg Albrecher and Jevgenijs Ivanovs. A risk model with an observer in a markov environment. *Risks*, 1(3):148–161, 2013.

[AI14] Hansjörg Albrecher and Jevgenijs Ivanovs. Power identities for lévy risk models under taxation and capital injections. *Stochastic Systems*, 4(1):157–172, 2014.

[AI15] Hansjoerg Albrecher and Jevgenijs Ivanovs. Strikingly simple identities relating exit problems for lévy processes under continuous and poisson observations. *arXiv preprint arXiv:1507.03848*, 2015.

[AIZ14] Hansjoerg Albrecher, Jevgenijs Ivanovs, and Xiaowen Zhou. Exit identities for levy processes observed at poisson arrival times. *arXiv preprint arXiv:1403.2854*, 2014.

[AKP04] Florin Avram, Andreas E Kyprianou, and Martijn R Pistorius. Exit problems for spectrally negative lévy processes and applications to (canadized) russian options. *The Annals of Applied Probability*, 14(1):215–238, 2004.

[AM14] Pablo Azcue and Nora Muler. *Stochastic Optimization in Insurance: A Dynamic Programming Approach*. Springer, 2014.

[AM15] Florin Avram and Andreea Minca. Steps towards a management toolkit for central branch risk networks, using rational approximations and matrix scale functions. In A. B. Piunovskyi, editor, *Modern trends in controlled stochastic processes: theory and applications*, page 263:285, 2015.

[AM16] Florin Avram and Andreea Minca. On the management of central branch risk networks. *Advances in Applied probability*, 2016.

[AMP16] Pablo Azcue, Nora Muler, and Zbigniew Palmowski. Optimal dividend payments for a two-dimensional insurance risk process. *arXiv preprint arXiv:1603.07019*, 2016.

[APP06] Florin Avram, M Pistorius, and Z Palmowski. A two-dimensional ruin problem on the positive quadrant, with exponential claims: Feynman-kac formula, laplace transform and its inversion. In *Ninth international conference Zaragoza-Pau on applied mathematics and statistics: Jaca (Spain), September 19-21, 2005*, pages 199–206. Seminario Matemático" García de Galdeano", 2006.

[APP07] Florin Avram, Zbigniew Palmowski, and Martijn R Pistorius. On the optimal dividend problem for a spectrally negative lévy process. *The Annals of Applied Probability*, pages 156–180, 2007.

[APP08a] Florin Avram, Zbigniew Palmowski, and Martijn Pistorius. A two-dimensional ruin problem on the positive quadrant. *Insurance: Mathematics and Economics*, 42(1):227–234, 2008.

[APP08b] Florin Avram, Zbigniew Palmowski, and Martijn R Pistorius. Exit problem of a two-dimensional risk process from the quadrant: exact and asymptotic results. *The Annals of Applied Probability*, 18(6):2421–2449, 2008.

[APP15] Florin Avram, Zbigniew Palmowski, and Martijn R Pistorius. On gerber–shiu functions and optimal dividend distribution for a lévy risk process in the presence of a penalty function. *The Annals of Applied Probability*, 25(4):1868–1935, 2015.

[APY16] F. Avram, J.L. Perez, and K. Yamazaki. First passage theory for parisian spectrally negative lévy processes regulated below by capital injections and reflected above. *Preprint*, 2016.

[BB11] Nicole Bäuerle and Anja Blatter. Optimal control and dependence modeling of insurance portfolios with lévy dynamics. *Insurance: Mathematics and Economics*, 48(3):398–405, 2011.

[BCR11] Andrei Badescu, Eric Cheung, and Landy Rabehasaina. A two-dimensional risk model with proportional reinsurance. *Journal of Applied Probability*, 48(3):749–765, 2011.

[Bog03] Elena Boguslavskaya. On optimization of dividend flow for a company in a presence of liquidation value. *Can be downloaded from http://www. boguslavsky. net/fin/dividendflow. pdf*, 2003.

[BPPR] EJ Baurdoux, JC Pardo, JL Pérez, and JF Renaud. Gerber-shiu distribution at parisian ruin for lévy insurance risk processes.

[DF57] Bruno De Finetti. Su un'impostazione alternativa della teoria collettiva del rischio. In *Transactions of the XVth international congress of Actuaries*, volume 2, pages 433–443, 1957.

[DW04] David Dickson and Howard R Waters. Some optimal dividends problems. *Astin Bulletin*, 34(01):49–74, 2004.

[Fro08] E Frostig. On ruin probability for a risk process perturbed by a lévy process with no negative jumps. *Stochastic Models*, 24(2):288–313, 2008.

[GLY06] Hans U Gerber, X Sheldon Lin, and Hailiang Yang. A note on the dividends-penalty identity and the optimal dividend barrier. *Astin Bulletin*, 36(02):489–503, 2006.

[IP12] Jevgenijs Ivanovs and Zbigniew Palmowski. Occupation densities in solving exit problems for markov additive processes and their reflections. *Stochastic Processes and their Applications*, 122(9):3342–3360, 2012.

[Iva11] Jevgenijs Ivanovs. *PhD thesis: One-sided Markov additive processes and related exit problems*. 2011.

[Iva13] Jevgenijs Ivanovs. Spectrally-negative Markov additive processes 1.0. https://sites.google.com/site/jevgenijsivanovs/files, 2013. Mathematica 8.0 package.

[KL10] A.E. Kyprianou and R. Loeffen. Refracted lévy processes. *Ann. Inst. H. Poincaré*, 46(1):24–44, 2010.

[KP08] Andreas E Kyprianou and Zbigniew Palmowski. Fluctuations of spectrally negative markov additive processes. In *Séminaire de probabilités XLI*, pages 121–135. Springer, 2008.

[KPP14] A Kyprianou, Juan Carlos Pardo, and José Luis Pérez. Occupation times of refracted lévy processes. *Journal of Theoretical Probability*, 27(4):1292–1315, 2014.

[Kyp14] Andreas E Kyprianou. *Fluctuations of Lévy Processes with Applications: Introductory Lectures*. Springer Science & Business Media, 2014.

[Loi05] Stéphane Loisel. Differentiation of some functionals of risk processes, and optimal reserve allocation. *Journal of applied probability*, pages 379–392, 2005.

[LR10] Ronnie L Loeffen and Jean-Franrcois Renaud. De finetti's optimal dividends problem with an affine penalty function at ruin. *Insurance: Mathematics and Economics*, 46(1):98–108, 2010.

[LRZ11] David Landriault, Jean-Franrcois Renaud, and Xiaowen Zhou. Occupation times of spectrally negative lévy processes with applications. *Stochastic processes and their applications*, 121(11):2629–2641, 2011.

[LST14] Gunther Leobacher, Michaela Szölgyenyi, and Stefan Thonhauser. Bayesian dividend optimization and finite time ruin probabilities. *Stochastic Models*, 30(2):216–249, 2014.

[LWS09] Bo Li, Rong Wu, and Min Song. A renewal jump-diffusion process with threshold dividend strategy. *Journal of Computational and Applied Mathematics*, 228(1):41–55, 2009.

[LZ08] Arne Løkka and Mihail Zervos. Optimal dividend and issuance of equity policies in the presence of proportional costs. *Insurance: Mathematics and Economics*, 42(3):954–961, 2008.

[MM61] Merton H Miller and Franco Modigliani. Dividend policy, growth, and the valuation of shares. *the Journal of Business*, 34(4):411–433, 1961.

[Pic94] Philippe Picard. On some measures of the severity of ruin in the classical poisson model. *Insurance: Mathematics and Economics*, 14(2):107–115, 1994.

[Pis03] MR Pistorius. On doubly reflected completely asymmetric lévy processes. *Stochastic Processes and their Applications*, 107(1):131–143, 2003.

[Pis04] MR Pistorius. On exit and ergodicity of the spectrally one-sided lévy process reflected at its infimum. *Journal of Theoretical Probability*, 17(1):183–220, 2004.

[Pis05] Martijn Pistorius. A potential-theoretical review of some exit problems of spectrally negative lévy processes. *Séminaire de Probabilités XXXVIII*, pages 30–41, 2005.

[PY15] José Luis Pérez and Kazutoshi Yamazaki. On the refracted-reflected spectrally negative lévy processes. *Preprint*, 2015.

[Ren14] Jean-Franrcois Renaud. On the time spent in the red by a refracted lévy risk process. *Journal of Applied Probability*, 51(4):1171–1188, 2014.

[RSST09] Tomasz Rolski, Hanspeter Schmidli, Volker Schmidt, and Jozef Teugels. *Stochastic processes for insurance and finance*, volume 505. John Wiley & Sons, 2009.

[Sat99] K.I. Sato. *Lévy processes and infinitely divisible distributions*. Cambridge university press, 1999.

[SLG84] Steven E Shreve, John P Lehoczky, and Donald P Gaver. Optimal consumption for general diffusions with absorbing and reflecting barriers. *SIAM Journal on Control and Optimization*, 22(1):55–75, 1984.

[ZYY13] Zhimin Zhang, Hailiang Yang, and Hu Yang. On a sparre andersen risk model perturbed by a spectrally negative levy process. *Scandinavian Actuarial Journal*, 2013(3):213–239, 2013.