Université Paris-Dauphine

Solving business problems with research

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Overview

1. Theory v. practice
2. About Longevitas
3. The longevity-risk problem
4. Multi-year view
5. Deferred annuities
6. VaR v. CTE
7. Managing longevity risk
8. Conclusions

1 Theory v. practice

Theory v. practice

"practice suggests problems essentially new for science and thus challenges one to seek quite new methods. And if theory gains much when new applications or new developments of old methods occur, the gain is still greater when new methods are discovered"

Chebyshev [1856]

2 About Longevitas

2 About Longevitas

- Founded 2006.
- Based in Edinburgh.
- Clients in UK, USA, Canada and Switzerland.
- Research partnership with Heriot-Watt University.

2 Services for actuaries

- Experience analysis and mis-estimation:

- Stochastic mortality projections and capital:

- Rating pension schemes:

3 The longevity-risk problem

3 Trend v. one-year view

"Whereas a catastrophe can occur in an instant, longevity risk takes decades to unfold"

The Economist [2012]

3 Trend v. one-year view

- Longevity trends emerge slowly over many years…

  …but insurance regulations view risks as single-year catastrophes.
- How do you reconcile the two?
- How do you fit a long-term risk into a short-term view?

3 Trend v. one-year view

Time for some Chebyshevian new methods:

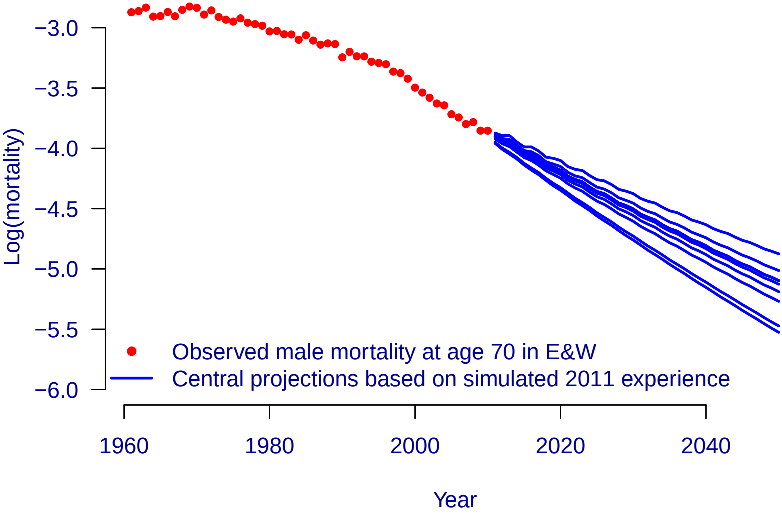
- Create new models just for this specific task, e.g Plat [2011] and Börger [2010], or
- Create a framework for existing projection models like Lee and Carter [1992], Cairns et al. [2006].

3 Trend v. one-year view

Solution from Richards et al. [2014]:

1. Pick a model and fit it to real data.
2. Use model to simulate next year's experience data.
3. Refit the model using real and simulated data.

3 Sensitivity of forecast



Source: Lee-Carter example from Richards et al. [2014].

3 Trend v. one-year view

Solution from Richards et al. [2014]:

1. Pick a model and fit it to real data.
2. Use model to simulate next year's experience data.
3. Refit the model using real and simulated data.
4. Calculate liability value, x, with new model.
5. Discard simulated experience data from (2).
6. Repeat (2)–(5) a few thousand times.

Sample of liability values $\{x_1, x_2, \ldots, x_m\}$.

3 Liability values

- Our unknown liability is X (say).
- VaR-style solvency capital:

$$\left(\frac{Q_\alpha}{\mathbb{E}[X]} - 1\right) * 100\%$$

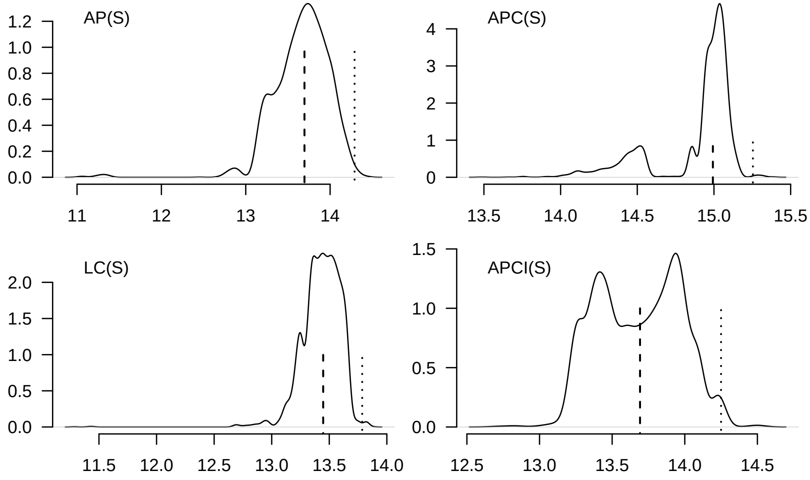
where Q_α is α-quantile of X, i.e. $\Pr(X < Q_\alpha) = \alpha$.

3 Simulated liability values

- We don't know the distribution of X ...

  ... but we do have a sample $\{x_1, x_2, \ldots, x_m\}$.
- Estimate $\mathbb{E}[X]$ from mean of sample.
- Estimate Q_α from sample using Harrell and Davis [1982].

3 One-year liability densities



Annuities payable to male aged 70. Means marked with dashed line and $Q_{99.5\%}$ marked with dotted line.
Source: Richards et al. [2017, Table 4].

3 Value-at-risk

- Wide variety of density shapes.

  ⇒ not all unimodal…

  … and not all symmetric.
- Considerable variability between models.

  ⇒ need to use multiple models…

  … and exercise *actuarial judgement*.

4 Multi-year view

4 Multi-year view

- Richards et al. [2014] was for one-year insurer solvency.
- The same methodology has other applications. . .

4 Multi-year view

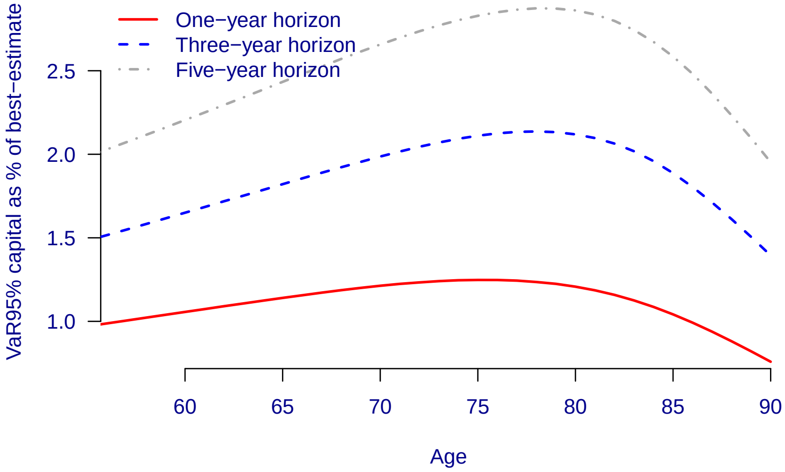
Medium-term business planning:

- 3–5 years for insurer ORSA.
- Ten-year “glide path” to buy-out for pension schemes.

4 Multi-year view

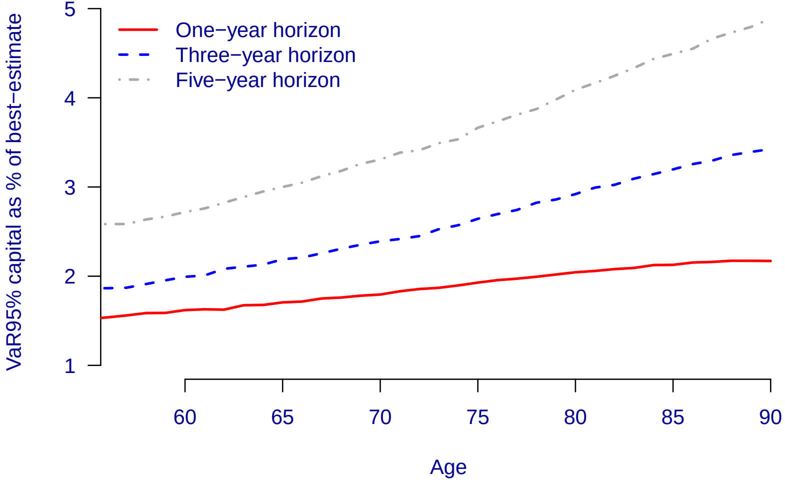
- Take one-year framework from Richards et al. [2014].
- Extend time horizon to 3–5 years.
- Reduce p-value to, say, 95%...

4 Females, Lee-Carter model



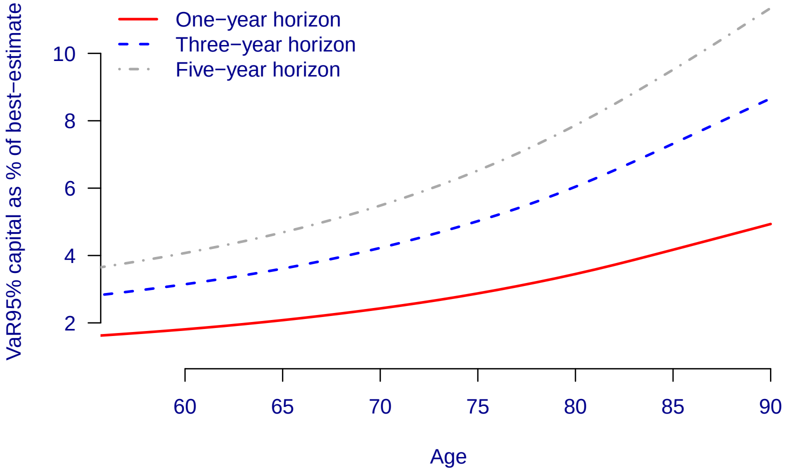
Immediate annuities under Lee-Carter model. UK data ages 50–104, 1971–2016

4 Females, APC model



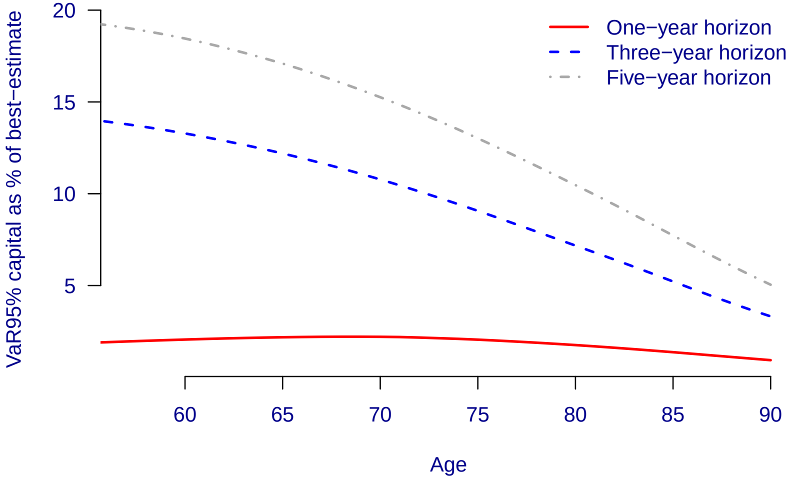
Immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016

4 Females, Cairns et al. [2006]



Immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016

4 Males, Lee-Carter model



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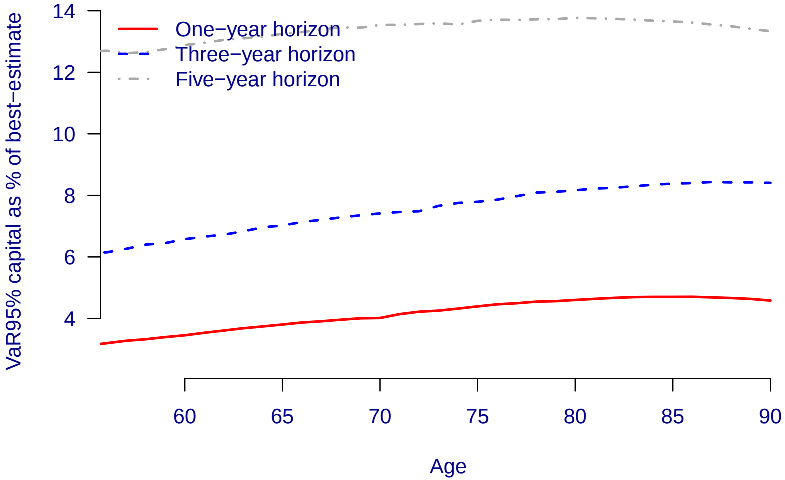
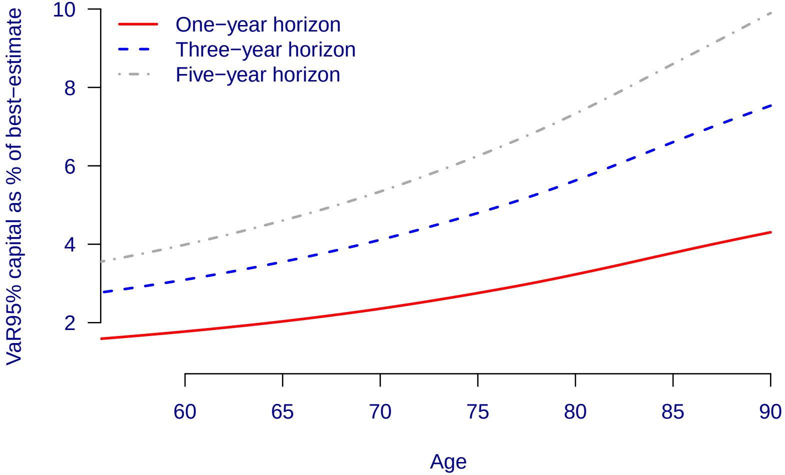


Immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016

4 Males, Cairns et al. [2006]



Immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016

4 Multi-year view

- Considerable variability between models, as with one-year view.
- No consistent pattern in capital by term.

$\Rightarrow$ need to use multiple models…

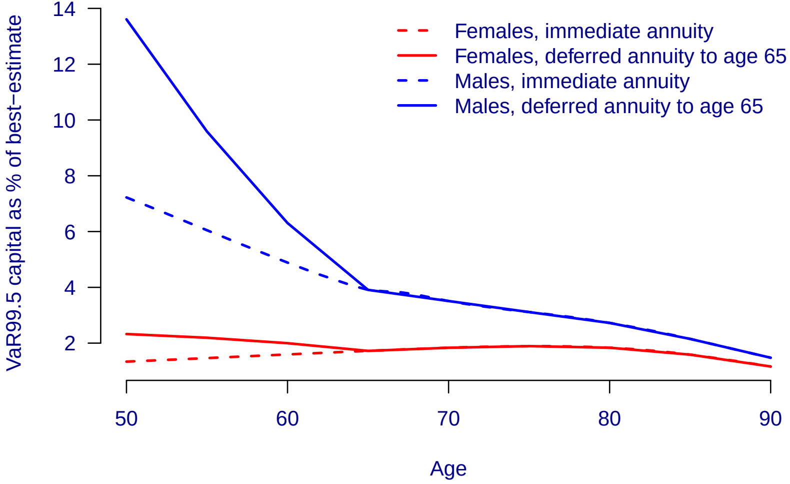
… and exercise actuarial judgement (*again!*).

5 Deferred annuities

5 Deferred annuities

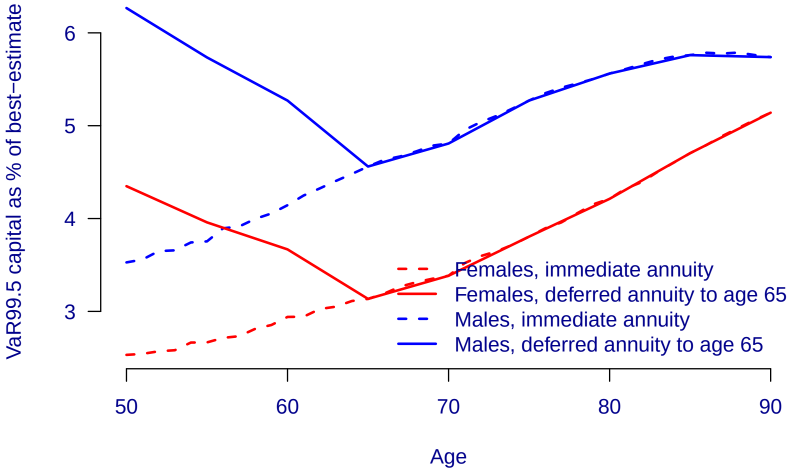
- Most published work concerns immediate annuities and pensions in payment.
- What about deferred annuities and pensions?
- Assume payment from age 65.
- Compare VaR99.5% solvency capital for immediate and deferred annuities.

5 Solvency capital, Lee-Carter



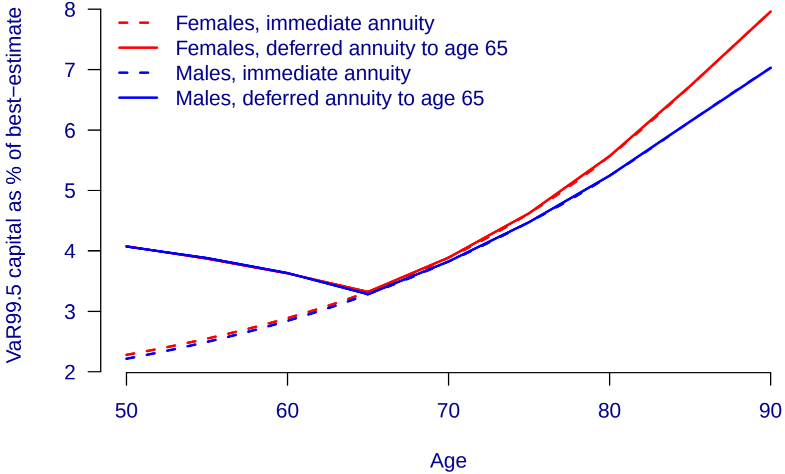
Deferred and immediate annuities under Lee-Carter model. UK data ages 50–104, 1971–2016

5 Solvency capital, APC model



Deferred and immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016

5 Solvency capital, Cairns et al



Deferred and immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016

5 Deferred annuities

- Depending on age, solvency capital for deferred annuities can be double that of annuities in payment.
- Sharp differences in solvency capital by gender.

6 VaR v. CTE

6 VaR

- Our unknown liability is X (say).
- VaR-style solvency capital:

$$\left(\frac{Q_\alpha}{\mathbb{E}[X]} - 1\right) * 100\%$$

where Q_α is α-quantile of X, i.e. $\Pr(X < Q_\alpha) = \alpha$.

6 CTE

- Our unknown liability is X (say).
- CTE-style solvency capital:

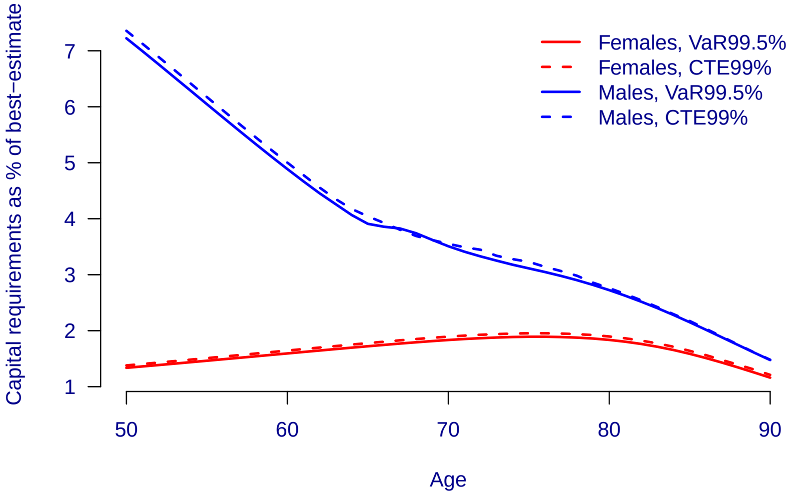
$$\left(\frac{\mathbb{E}[X|X \geq Q_\alpha]}{\mathbb{E}[X]} - 1\right) * 100\%$$

where Q_α is α-quantile of X, i.e. $\Pr(X < Q_\alpha) = \alpha$.

6 VaR v. CTE

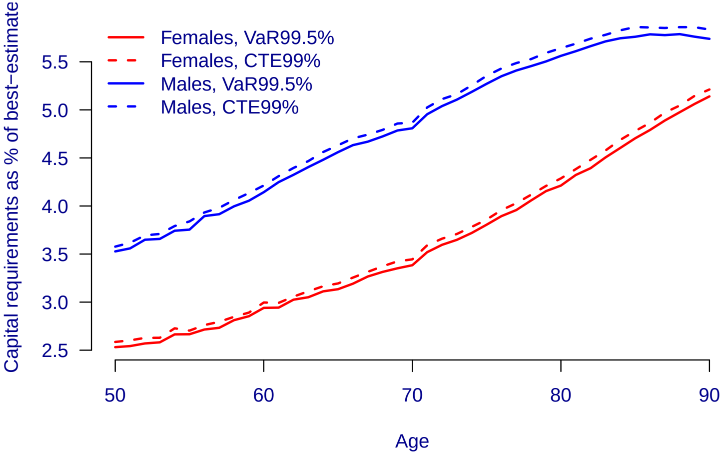
- How does VaR capital compare to CTE capital?
- $\text{CTE}_\alpha > \text{VaR}_\alpha$ (obviously!)
- But how does VaR99.5% compare to CTE99%?
- Can calculate both from same sample...

6 UK, Lee-Carter model



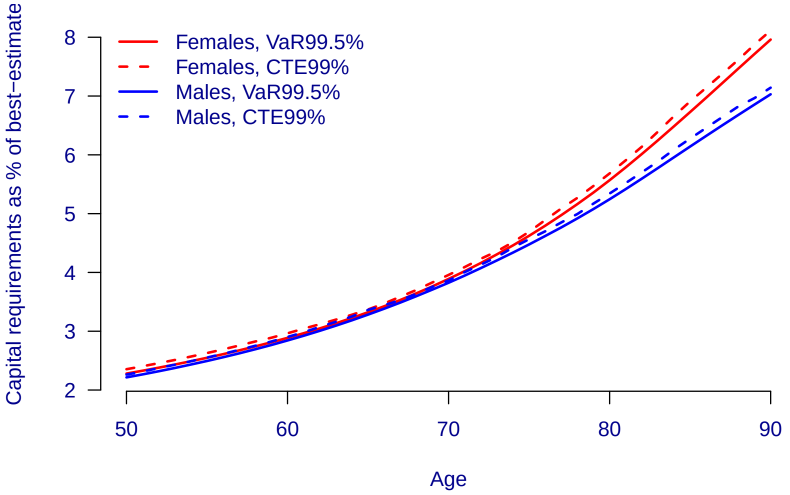
Annuities in payment under Lee-Carter model. UK data ages 50–104, 1971–2016

6 UK, APC model



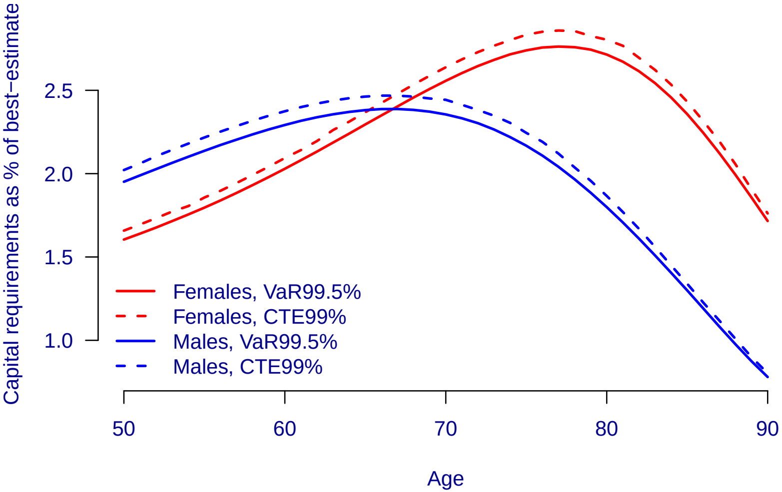
Annuities in payment under APC(S) model. UK data ages 50–104, 1971–2016

6 UK, Cairns et al. [2006]



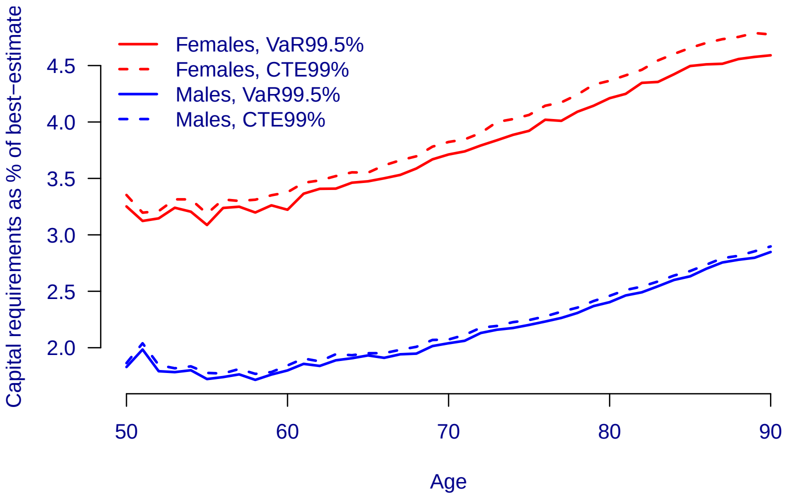
Annuities in payment under M5(S) model. UK data ages 50–104, 1971–2016

6 Netherlands, Lee-Carter



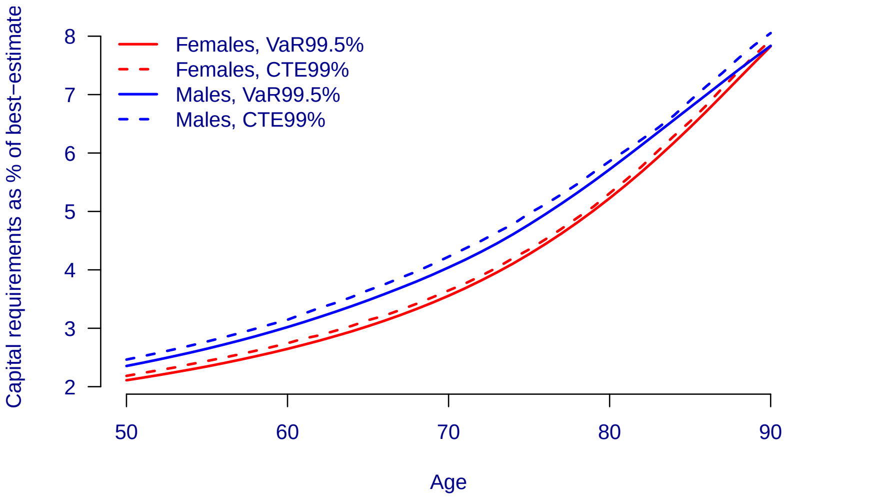
Annuities in payment under LC(S) model. Netherlands data ages 50–104, 1971–2016

6 Netherlands, APC model



Annuities in payment under APC(S) model. Netherlands data ages 50–104, 1971–2016

6 Netherlands, Cairns et al. [2019]



Annuities in payment under M5(S) model. Netherlands data ages 50–104, 1971–2016

6 VaR v. CTE

- Longevity trend-risk capital very comparable between VaR99.5% and CTE99%.
- CTE99% usually slightly more prudent than VaR99.5%.
- Difference usually under 0.1%.

7 Managing longevity risk

7 Managing longevity risk

- Keep risk, or
- Transfer risk, or
- Hedge risk.

7 Transferring longevity risk

- Insurers historically used indemnity reinsurance to manage risk.
- "Indemnity" means the exact portfolio experience is insured.
- The insurer is left with no longevity risk...

  ...although there is a risk that the reinsurer might fail (counterparty risk).

7 Hedging longevity risk

- However, insurers can also use *hedging contracts*.
- The value of a hedging contract is supposed to move in line with the liabiities.
- The insurer is left with less longevity risk (hopefully)...

  ...although there is a risk that the hedge is imperfect (basis risk).
- How big is this risk?

7 Index-based hedges

- Define contract using population mortality.
- Term n years.
- At end of term, fit Lee-Carter model (say) and it use to value annuity with unknown value X.
- Use a function of X to close out the contract.

  $\Rightarrow$ This is just another multi-year VaR calculation.

7 Index-based hedges

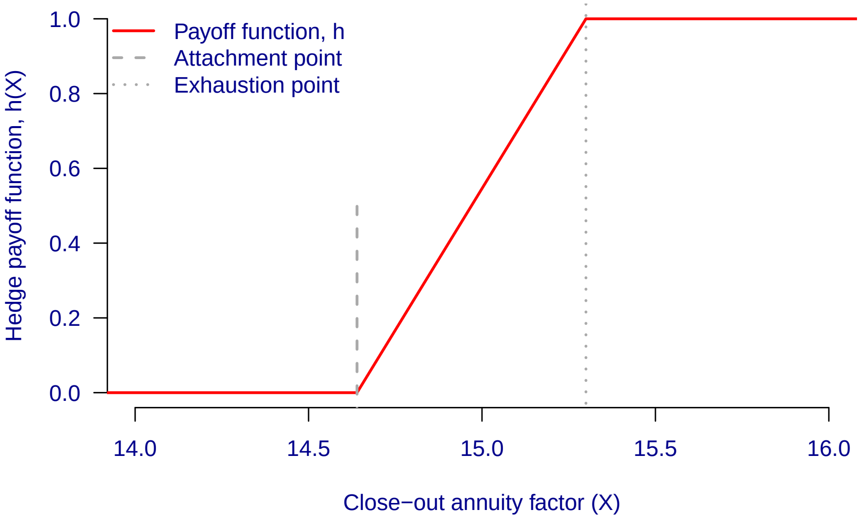
LONGEVITAS

- Risk metric (annuity value) is X.
- Only pay above attachment point, AP.
- Pay no more than exhaustion point, EP.
- Standardise payoff, h, as:

$$h(X) = \max\left(0, \min\left(\frac{X - AP}{EP - AP}, 1\right)\right)$$

- See Cairns and El Boukfaoui [2017] for detailed discussion.

7 Hedge payoff function



7 Index-based hedges

- Set $AP = Q_{\alpha_1}$ and $EP = Q_{\alpha_2}$ $(\alpha_1 < \alpha_2)$.
- Q_α set with reference to Lee-Carter sample paths over n years, i.e. an n-year VaR simulation.
- Probability of payoff is $1 - \alpha_1$.
- Mean payoff can be estimated from VaR results.

7 Example hedge contract

- $n = 15$ years.
- Use Lee-Carter model for close-out calculation.
- Follow Cairns and El Boukfaoui [2017] and set $AP = Q_{60\%}$ and $EP = Q_{95\%}$.
- Probability of a payoff is 0.4.
- Average payoff is 0.375 (from 5,000 simulations).

7 Model risk

- Lee-Carter model used for both sample paths over n years **and** for payoff calculation.
- Assume we keep the Lee-Carter model for payoff calculation and also keep the same attachement and exhaustion points.
- What happens if the sample paths follow a *different* model?

7 Hedge assessment

Impact of different sample-path models on payoff:

Model	Payoff prob.	Mean payoff
LC(S)	0.40	0.375
M5(S)	0.53	0.592
2DAC	0.80	0.434
M6	0.82	0.710

Source: own calculations using population data for males in Netherlands, ages 50–104, 1971–2016.
Annuity values discounted at 2% p.a.

7 Hedge assessment

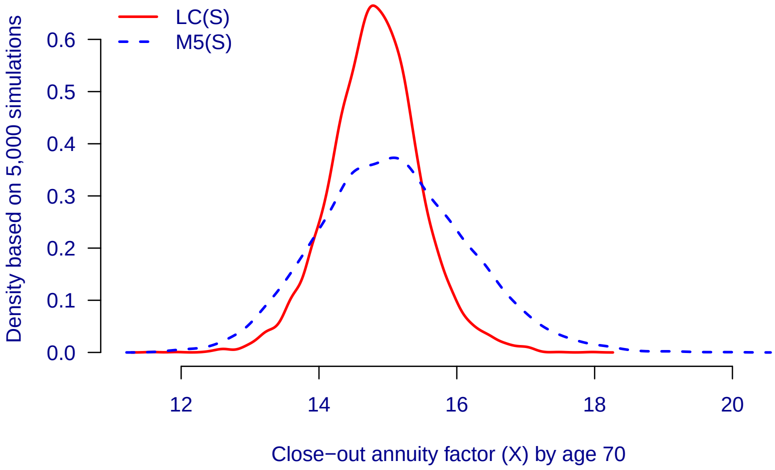
- Model for future mortality is unknowable (model risk).
- So payoff probability and expected payoff are also unknowable.
  - What value should the hedge contract have on the balance sheet?
  - What solvency capital relief should be given?

$\Rightarrow$ Actuarial judgement required on both counts.

7 Hedge assessment

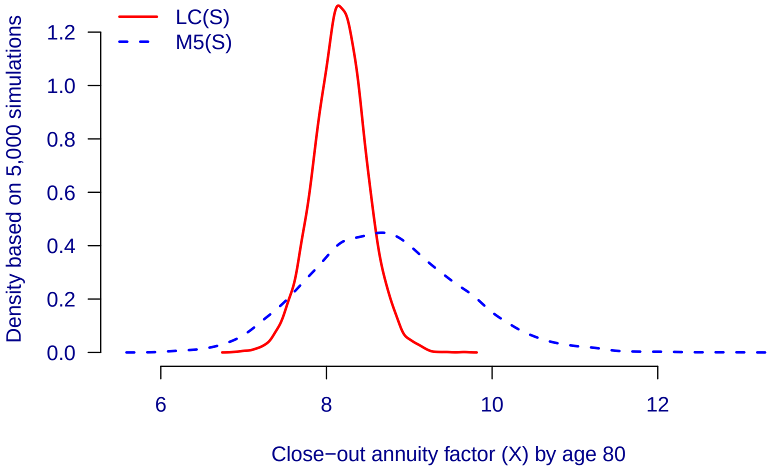
- How different can the answers get?
- Consider the spread at various ages under CBD model (M5)...

7 M5 sample paths — age 70



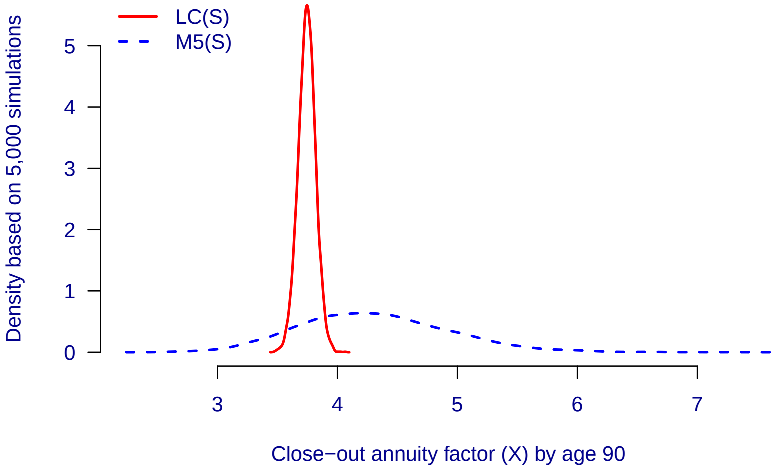
VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model. Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.

7 M5 sample paths — age 80



VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model. Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.

7 M5 sample paths — age 90



VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model. Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.

8 Conclusions

8 Conclusions

- Longevity trend risk can be put into a one-year framework.
- Same outputs can be used for both VaR- and CTE-style solvency regimes.
- Framework extends to ORSA for insurers…

  ….and "glide paths" to buy-outs

  ….and assessing index-based hedges.
- Model risk is critical throughout.
- Expert judgement required for solvency capital…

  ….and valuation of index-based hedges.

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More on longevity risk at www.longevitas.co.uk